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Review

Large Eddy simulation of turbulent flow in Rayleigh-Bénard configuration

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Abstract

This work undertakes a numerical study of turbulent incompressible flows in Rayleigh-Benard configurations using Large Eddy Simulation and two sub-grid scale models, i.e., the WALE (Wall Adapting Local Eddy-viscosity) model and the corresponding dynamic sub-grid model (DSGS). In the process of using DSGS, an optimal value of constant C_W of the WALE model was determined for the envisaged Rayleigh number (Ra=6.3.10^5). The computed numerical results (on a relatively coarse grid) showed good agreement with those Direct Numerical Simulation (DNS) results found in the literature. It is observed that the profiles obtained are highly dependent on the time interval over which the statistics are made (equivalent to the Interval of Statistical Analysis (ISA).

Keywords: Turbulent natural convection, Finite volume method, Large eddy simulation.

INTRODUCTION

Turbulent natural convection with two differentially heated side walls has been an attractive subject in fundamental turbulence research. In this work we will start by presenting the equations governing the incompressible flows of fluids under the approximations of Boussinesq, then will present the filtered equations related to these flows. The turbulent eddy viscosity appearing in the filtered conservation equations of the momentum is modeled by the WALE model of Nicoud (Nicoud and Ducro, 1999). For the turbulent thermal diffusivity, a simple relation between turbulent kinematic viscosity and turbulent Prandtl number were used. The validation of the computer code was carried out by a problem of turbulent natural convection between two infinite plans subjected to different temperatures (Rayleigh-Bénard problem).

It is well known that for high Rayleigh number turbulent flows, the extended range of turbulent scales is of concern. Direct Numerical Simulation (DNS) cannot solve high Rayleigh turbulent flows due to the large amount of computational information generated by the large range of scales. Owing to this drawback, DNS is normally restricted to low and moderate Reynolds number flows.

The Large Eddy Simulation (LES) methodology introduced by Smagorinsky (1963) is situated between highest degrees of DNS and RANS (for Reynolds Averaged Navier–Stokes). LES is expressed by the partition of the large eddies structures and sub-grid scales structures using a grid filter. Large-scale flow motions are explicitly computed, while small-scale flow motions are modeled with a sub-grid scale (SGS) model. LES is superior to DNS in terms of computational cost, and better than RANS in terms of accuracy and data availability.

Several SGS models have been utilized by a group of researchers: the Smagorinsky's model (Smagorinsky, 1963), the dynamic SGS model initially proposed by Germano et al., (1991) or the WALE model developed by Nicoud and Ducros (1999).

In this work, we study the case of turbulent natural convection flows between two infinite plans subjected to different temperatures. The adopted methodology is based on the finite volume method, coupled with a full-multigrid acceleration and LES. A computational code to simulate transient, incompressible, three-dimensional flows was developed (Ben-Beya, 1995; Ben-Cheikh, 2008) using the projection method (Achdou and Guermond, 2000).
Herein, two different SGS models were implemented, namely the WALE model (Nicoud and Ducros,1999) and the

corresponding dynamic model (DSGS) (Sagaut, 2001).

The three-dimensional flow in a natural convective flow in a differentially heated cavity has been studied experimentally (Benkhalifa and Penot, 2006; Valencia et al., 2007) and numerically (Mc Laughlin and Orszag, 1982; Hongxing and Zuojin, 2006; Shetty et al., 2010). Statistical studies on the mean velocities, Nusselt number are performed and compared with those obtained numerically by other authors.

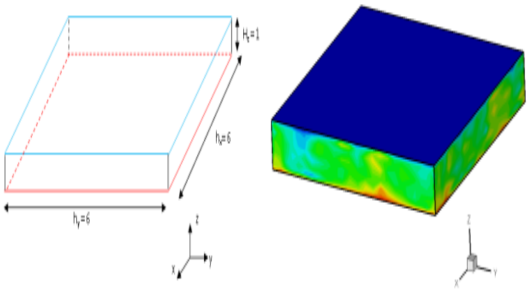

Figure1. Geometry of turbulent Rayleigh-Bénard convection problem with Aspect ratio (6:6:1) (left) and the representation of the isotherms for Ra=6.3x10^5(right)

Governing Equations

The unsteady Navier-Stokes equations for incompressible flows are:

$$\frac{\partial u_i}{\partial t}=0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial\left(u_i u_j\right)}{\partial x_j}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_j \partial x_j}+g\beta\left(T-T_r\right)\delta_{i3} \qquad (2)$$

$$\frac{\partial T}{\partial t}+\frac{\partial\left(Tu_j\right)}{\partial x_j}=\alpha\frac{\partial^2 T}{\partial x_j \partial x_j} \qquad (3)$$

where $\alpha=\frac{\lambda}{\rho_0 C_p}$ is the thermal diffusivity.

The LES equations are obtained by applying a filtering operation of where the length of cut is Δ, the filtered dimensional equations become:

$$\frac{\partial \bar{u}_i}{\partial t}=0 \qquad (4)$$

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial\left(\bar{u}_i \bar{u}_j\right)}{\partial x_j}=-\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x_i}+\nu\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}-\frac{\partial \tau_{ij}}{\partial x_j}+g\beta\left(\bar{T}-T_r\right)\delta_{i3} \qquad (5)$$

Where the sub-grid scale stresses are given by:

$$\tau_{ij}=-\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)+\frac{1}{3}\tau_{kk}\delta_{ij} \qquad (6)$$

The filtered energy equation is:

$$\frac{\partial \bar{T}}{\partial t}+\frac{\partial\left(\overline{Tu_j}\right)}{\partial x_j}=\alpha\frac{\partial^2 \bar{T}}{\partial x_j \partial x_j} \qquad (7)$$

$$\frac{\partial \bar{T}}{\partial t}+\frac{\partial\left(\bar{T}\bar{u}_j\right)}{\partial x_j}=\alpha\frac{\partial^2 \bar{T}}{\partial x_j \partial x_j}-\frac{\partial h_j}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\alpha\frac{\partial \bar{T}}{\partial x_j}-h_j\right) \qquad (8)$$

The term h_j is modelled by:

$$h_j = -\alpha_t \frac{\partial \overline{T}}{\partial x_j} \quad (9)$$

The modeling of the term of turbulent thermal diffusivity is generally done by the intermediary of a turbulent Prandtl number. Within the framework of this work we chose $\Pr_t = 0.9$ in the relation: $\alpha_t = \frac{\nu_t}{\Pr_t}$.

Numerical procedure

The unsteady Navier-Stokes equations are discretized using staggered, non-uniform control volumes. A projection method attributed to Achdou and Guermond (2000) is used to adequately couple the momentum, continuity and energy equations.
The finite-volume method developed by Patankar and Spalding (1972) is employed to discretize the Navier–Stokes equations. The Poisson pressure correction equation is solved using a full multi-grid method as suggested by Ben-Cheikh (2008). The numerical methodology was implemented with a FORTRAN program.

RESULTS AND DISCUSSION

In order to validate the computer code, the problem of turbulent natural convection between two infinite plans was retained. Figure1 illustrates the situation of the problem.
The boundary conditions on the two vertical walls can be written as follows:

$$u=0, v=0, w=0, \left(\frac{\partial \theta}{\partial x}\right)=0, \text{ for } x=0, y\in[0,6], z\in[0,1].$$

$$u=0, v=0, w=0, \left(\frac{\partial \theta}{\partial x}\right)=6, \text{ for } x=0, y\in[0,6], z\in[0,1].$$

For the lower wall, the boundary conditions are:

$$u=0, v=0, w=0, \theta=1, \text{ for } x\in[0,1], y\in[0,6], z=0.$$

For the higher wall, the boundary conditions are:

$$u=0, v=0, w=0, \theta=0, \text{ for } x\in[0,1], y\in[0,6], z=1.$$

In order to maximize the tests and to study the effect of some parameters on the quality of the results, we carried out simulations on a very coarse grid of dimension (24.24.8). The grid is with variable steps in the vertical direction and with constant steps in the other directions. On the basis of an initial zero field in all the fields, calculations were launched for Ra=6.3.10^5 and Pr=0.71.

In a first stage ($0<t<400$) the equations were solved without calling upon any SGS model. Figure 2 represents the evolution of the component speed *u* at a point of co-ordinate (0.5,0.5,0.7) according to a dimensional time t.

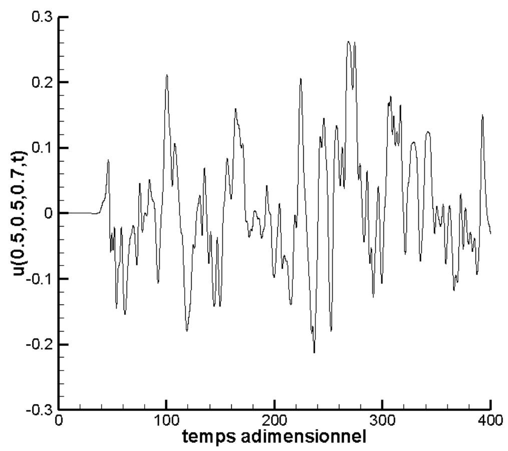

Figure 2: temporal evolution of the component speed *u* at the point of co-ordinates (0.5, 0.5,0.7).

The evolution of this component shows that after a dimensional time of approximately 50, the flow evolves to a

strongly non stationary state with presence of several frequencies.
The results obtained at $t = 400$ were used thereafter as initial field to evaluate the value of constant C_W of the LES model using the DSGS model. Figure3shows the time evolution of the space averaged value of C_W over the computational domain. The time averaged value of $\langle C_W \rangle$ in the interval $400 \le t \le 600$ gave a value of an optimal WALE constant $C_W^{opt} = 0.205$.

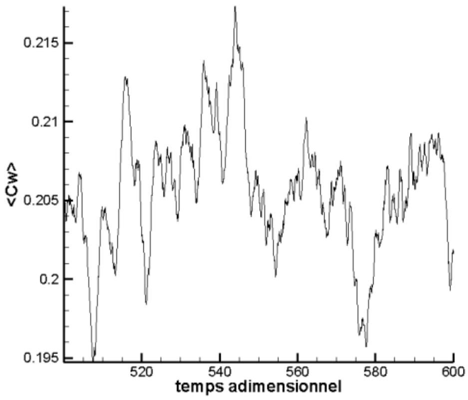

Figure3. Time evolution of the space averaged coefficient C_W

At this stage, calculations were continued from the field obtained at $t = 400$ with the SGS model and C_W=0.205. In order to let the flow converge towards an established transitory state, the calculations was launched over a time of integration $400 \le t \le 800$. Statistical calculations were then carried out on three intervals of different times, i.e., $800 \le t \le 1800$, $1800 \le t \le 2800$ and $2800 \le t \le 3800$. Figure 4 shows the evolution of the component *u*(0.5,0.5,0.7,t) and the square of that component u^2(0.5,0.5,0.7,t) on the interval of time $1800 \le t \le 2800$. It shows the strongly non stationary character of the flow. The storage of the quantities *u* and squared *u* is in fact necessary to make possible the statistical calculations presented in the next paragraph.

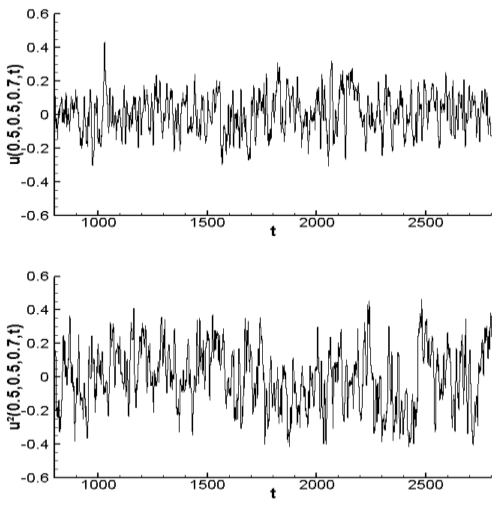

Figure 4. Time evolution of the component speed u and its square u^2 at the point of co-ordinates (0.5, 0.5,0.7)

Statistical calculations were carried out in of x=y=0.5 and each point Z of the intersection of these two plans (midline). With regard to the average quantities we used the following relation:

$\langle X \rangle = \sum_{i=1}^{K\max} X_i$ where X indicates one of the components u, v, w speed or the dimensionless temperature θ. K_{max} is the total number of iterations of the integration interval. Regarding the average fluctuating quantities, it has been determined using the classical relation:

$\langle f'g' \rangle = \langle fg \rangle - \langle f \rangle . \langle g \rangle$

Thus, the standard deviations of the fluctuating temperature θ usually noted θrms is defined by:

$$\theta rms = \left(\langle \theta^2(z) \rangle - \langle \theta(z) \rangle . \langle \theta(z) \rangle \right)^{1/2} \quad (10)$$

On figure 5 and 6 we report the quantities $\langle \theta \rangle$, and θrms along the midline. Sight the coarse size of the grid that we used, we can make only one qualitative comparison between the results obtained and those of the literature.

It appears clearly that the statistical results strongly depend on the total time of integration of the equations. More time of sampling is large plus the results convergent towards the DNS solution.

This tendency is also observed in the profile of θrms (fig. 6). Indeed, for $t = 300$, the LES results are much closer to the DNS results than for $t \leq 2000$. It is seen thus, that in spite of the coarse size of the grid used, the results are promising as well with regard to the average quantities the fluctuating quantities.

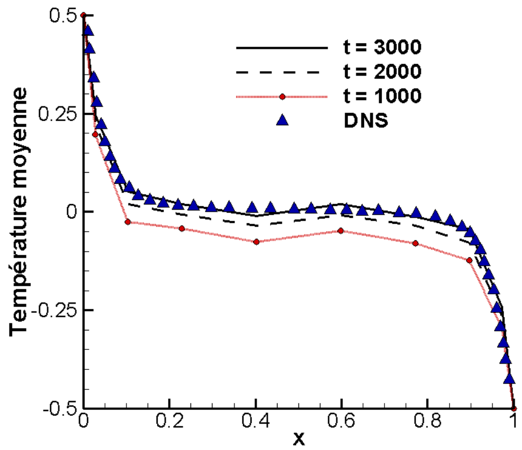

Figure5. Representation of the average temperature θ_{rms} along the line of centers for various adimensional times, comparison with calculations DNS.

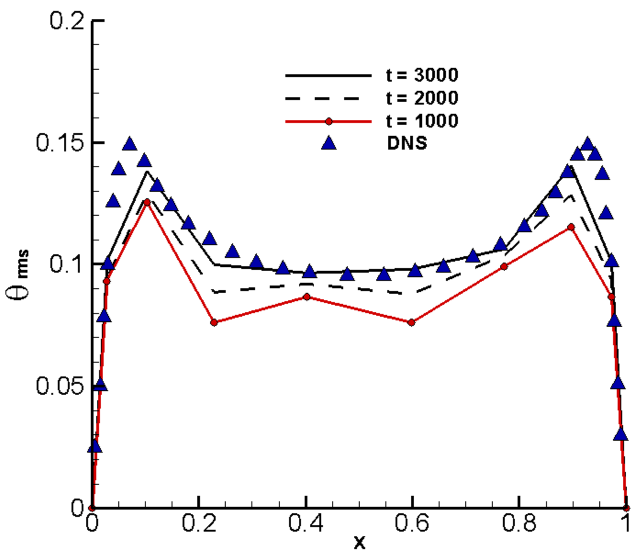

Figure 6. Representation of the average temperature $\langle \theta \rangle$ along the line of centers for various dimensional times, comparison with calculations DNS

Isotherms for Ra=6.3x10^5 are presented in figure 1It shows the turbulent character of the flow as well as a distribution of typical temperature to the strongly non stationary flows.

The average transfer of heat through the active walls characterized by the number of Nusselt and defines by: $\overline{Nu} = \frac{1}{S} \int_S Nu\, dS$

With $dS = dx.dy$

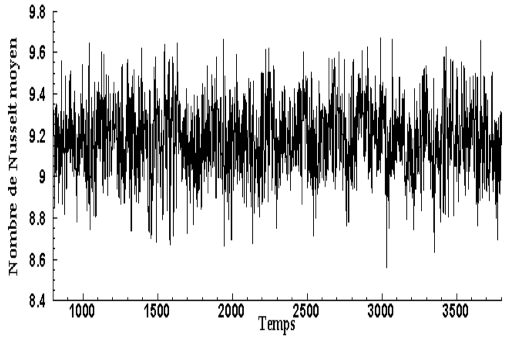

Figure7. shows the evolution of this number in the course of time

The averaged value of the Nusselt numbering time is then about 9.18. Since the value of this number is 7.27with a DNS calculation (Woerner, 1986), we deduce that the gap is 26.3%. This difference is quite normal since the used grid is very coarse (24.24.8).It should be noted that the calculation of the reference DNS (Woerner, 1986) were carried out on a grid ofdimension200.200.49. Thus, this study shows that a coarse grid allows a qualitative analysis of the correct flow. However, further quantitative analysis would necessity a finer computational grid points than the one we used.

CONCLUSIONS

The three dimensional classical problem of turbulent flows in Rayleigh-Benard configuration were simulated with a finite volume Large Eddy Simulation methodology in this work. Two sub-grid scale models were implemented, the WALE's (SGS) and the corresponding eddy viscosity dynamic model (DSGS). The results with the SGS model are very coherent with numerical data from other authors if an appropriate model constant C_w is accounted for. An optimal value of C_W were obtained by averaging in time the stored data of the space averaged values of $\langle C_W \rangle$ calculated with the DSGS model. For Ra=6.3.10^5, an optimal value $C_W^{opt} \approx 0.205$ was determined. It was also observed that the mean temperature component and the fluctuating quantities are extremely sensitive to the total simulation time or to the intervals of the statistical analysis (ISA).

The presented methodology in this work will be extended for numerical simulations using the SGS model on finer grids.

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