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Research Article

Parametric and self–excited vibrations induced by friction in a system with three degrees of freedom

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Abstract

The paper presents the analysis of a nonlinear parametric system consisting of a rotor with rectangular cross–section placed in a rigid self–excited base. The parametric instability zones have identified on the basis of the method of expanding into a power series in relation to two perturbation parameters (one connected with parametric excitation, the other with friction coefficient). The influence of the changes of chosen parameters of the system on the size and instability zones of the first order has been investigated.

Keywords: Rotor, Self–Excited and Parametric Vibration, Perturbation Parameters

INTRODUCTION

Friction induced self–excited vibrations and parametric vibrations occur in many physical systems and have been in the focus of interest for a long time in many works concerning vibrations (Stoker, 2010; Minorsky, 2011; Hayashi, 2011; Cunningham, 2011). Both kinds of vibrations may be considered as sufficiently known. However, when both excitations occur simultaneously in one system, the phenomenon is more complex (see for example, Alifov and Frolov; 2012). On the other hand, this case occurs in technology, because e.g. in the combustion engine, in certain conditions self–excited vibrations of the piston and parametrically excited vibrations together with forced vibrations are analyzed in this paper. The parametric excitation and the exciting force come from the rotor with rectangular cross–section, which has in its middle a cylinder-like mass concentrated eccentrically on it. The rotor is fixed on a base placed on a belt moving at constant velocity. At a certain value of the belt velocity and the frequency of rotor turns, parametric and self–excited vibrations are created in addition to the forced vibrations.

As the parametric excitation μ and the friction coefficient ϵ are small in such in such a system they have been recognized as perturbation parameters. The methods with one perturbation parameter used to determine the limits of the stability–loss zones are widely described in the literature, and their extensive presentation is given by Malkin, 2010; Giacaglia, 2010; Lakubovic and Starzinsky, 2013. However, the analytical approach based on introducing of two independent perturbation parameters is rarely used in mechanics. This paper present a general analytical technique for calculating the limits of stability in the system with self excited and parametric vibrations and develops authors earlier works (Awrejcewicz, 2010, 2011).

METHOD

The Analyzed System and Equations of Motion

The diagram of the analyzed system is presented in Figure 1. A weightless shaft with rectangular cross–section with a cylinder–like mass concentrated in its center is supported in the base placed on a belt moving at constant velocity V_0. The friction coefficient between the belt and the base depends on their relative velocity. The character of this

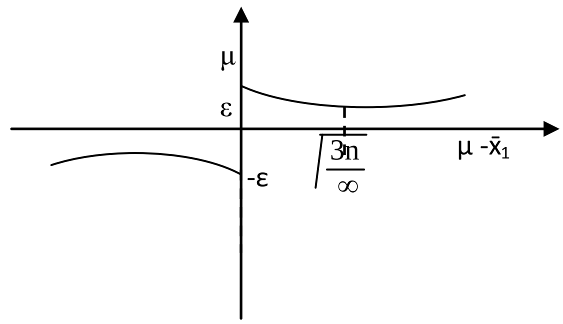

Figure 1. Diagram of the analyzed system

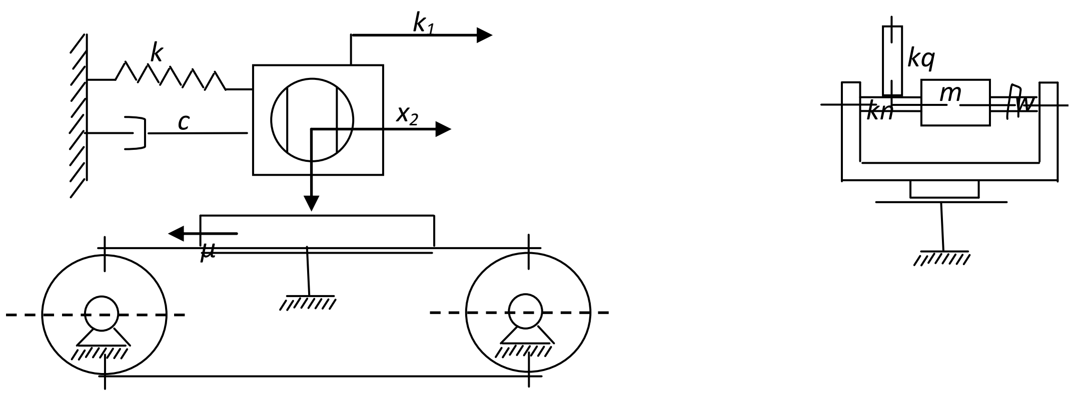

Figure 2. Dependence of the friction coefficient on the relative velocity

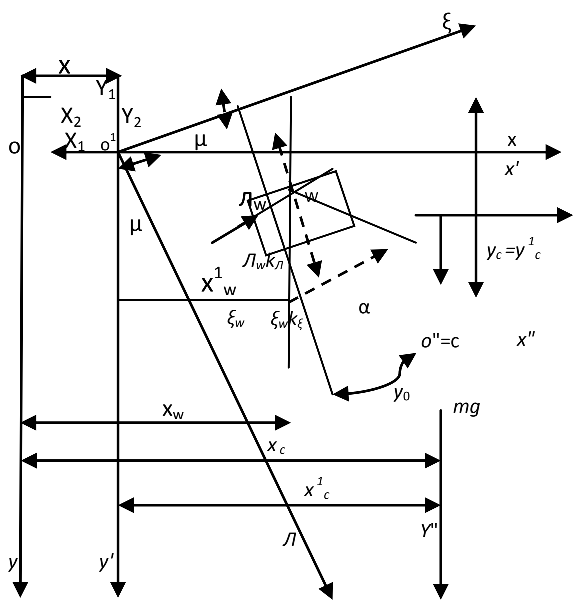

Figure 3. Calculation model of the system

dependence (Figure 2) causes the creation of self–excited vibrations. The effect is described in the basic works on nonlinear vibrations. On the other hand, considering the non–identical cross–section of the rotor at same values of its rotational speed, parametric vibrations occur. The vibrations cause the changes of the normal force holding down the base to the belt in vertical direction, and hence they cause the changes of the friction force. It is assumed that the vibration of the rotor does not cause the tearing of the base off the belt.

The calculation model of the analyzed system is presented in Figure 3. The equations of motion of the system have the form;

$m\ddot{x}_c = -\xi_w k_\xi \cos\varphi - \eta_w k_\eta \sin\varphi$
$m\ddot{y}_c = \xi_w k_\xi \sin\varphi + \eta_w k_\eta \cos\varphi + mg$
$I_{2''}\ddot{\varnothing} = -M_o + a(-\xi_w k_\xi \cos\varphi_o + \eta_w k_\eta \sin\varphi_o)$ (1)

Where;
$x_c\ y_c$: Coordinates of the centre of mass of the cylinder,
$I_{2''}$: Mass moment of the inertia of a cylinder with mass $_m$ in relation to the z" axis of the *O" x" y" z"* system moving with translator motion in relation to *Oxyz*
ξ_w, η_w. : Coordinates of the point of puncture by the shaft in the coordinates system $o' \xi n$
$o' \xi\eta$: Coordinates system whose axes are parallel to the main, central inertia axes of the cross section of the shaft.

k_ξ, k_η : Shaft rigidities *in the direction of the axes ξ and η*
M_o : Driving torque reduced by all the resistance torques
A, Φ_o : Parameters characterizing the position of the centre of mass of the disk C in relation to the point of puncture by the shaft.
For the state near the steady ones the torque M_o is very small. Let
$I_{z''} = m i_s^2$ (2)
Where i_s is the inertia radius, then the third equation of the Equation. (1) will assume the form

$$\ddot{\varnothing} = \frac{1}{m} - \frac{a}{i_s^2}\left(-\xi_w k_\xi \cos\varphi_o + \eta_w k_\lambda \sin\varphi_o\right) \quad (3)$$

As the eccentricity a and the shaft deflection ξ_w and n_w are small as compared to the inertia radius, and then the following can be assumed:
$\ddot{\varnothing} = 0,\ \dot{\varnothing} = w == \text{const},\ \varphi = wt$ (4)

The following geometric dependences result from the Fig 3:
$\xi_w = (x_w - x)\cos\varphi - y_w \sin\varphi$
$n_w = (x_w - x)\sin\varphi - y_w \cos\varphi$
$y_c = y_w + a\cos(\varphi + \varphi_o)$
$x_c = x_w + a\sin(\varphi + \varphi_o)$ (5)
where x_w, y_w are the coordinates of the point of puncture by the shaft *W* in the system *Oxy.*

In order to write down the equations of motion of the mass *M* it is necessary to determine the dynamic reactions on the shaft in its points of support. They are determined from the equations of equilibrium.

$X_1 + X_2 + \xi_w k_\xi \cos\omega t + \eta_w k_n \sin\omega t = 0$
$Y_1 + Y_2 + \xi_w k_\xi \sin\omega t + \eta_w k_n \cos\omega t = 0$ (6)
Where X_1, Y_1 and X_2, Y_2 denote the support reactions on the left and right end of the shaft, respectively. The rotor reactions on the support are then as follows

$R_x = -X_1, X_2$
$R_y = -Y_1, Y_2$ (7)
The equation of motion of a body with mass *M*, on the assumption that $M_g + R_y > 0$, has the form

$M\ddot{x} = -kx - c\dot{x} + R_x + (M_g + R_y)\,\mu(\omega),$
$\omega = v_o - \dot{x}$ (8)

The dependence of the friction coefficient on the relative velocity *w* can be circumscribed with the polynomial

$w = \varepsilon \operatorname{sgn}\omega - \alpha\omega + \beta\omega^3$ (9)

Finally, the equations of motion of the analyzed system, after assuming that $x = x_1$, $x_w = x_2$, $y_w = x_3$, have the form:

$\ddot{x}_1 = - x_1 [\Omega^2 + \Omega^2_\xi + \Omega^2_\eta + (\Omega^2_\xi - \Omega^2_\eta) \cos 2\omega t] - H\dot{x}_1 + x_2 [-(\Omega^2_\xi - \Omega^2_\eta) + - (\Omega^2_\xi - \Omega^2_\eta) \cos 2\omega t] - x_3 (\Omega^2_\xi - \Omega^2_\eta) \sin 2\omega t + \{g - x_2 (\Omega^2_\xi - \Omega^2_\eta) \sin 2\omega t + - x_3[-(\Omega^2_\xi - \Omega^2_\eta) + (\Omega^2_\xi - \Omega^2_\eta) \cos 2\omega t] + x_1 (\Omega^2_\xi - \Omega^2_\eta) \sin 2\omega t\}. [\varepsilon \sin (v_o e \dot{x}_1) - \alpha(v_o - \dot{x}_1) + \beta(v_o - \dot{x}_1)^3]$ (10)

$\ddot{x}_2 - x_2 [\omega^2_\xi - \omega^2_\eta + (\omega^2_\xi - \omega^2_\eta) \cos 2\omega t] - x_2 [\omega^2_\xi - \omega^2_\eta + (\omega^2_\xi - \omega^2_\eta)\cos 2\omega t]$
$+ x_3 [(\omega^2_\xi - \omega^2_\eta)\sin 2\omega t + \alpha^2\omega \sin(wt + \varphi_o)$
$\ddot{x}_3 = - x_1 (\omega^2_\xi - \omega^2_\eta) \sin 2\omega t + x_2(\omega^2_\xi - \omega^2_\eta) \sin 2\omega t + x_3[-(\omega^2_\xi - \omega^2_\eta) + (\omega^2_\xi - \omega^2_\eta). \cos 2\omega t] + \alpha\omega^2 \cos(\omega t + \varphi_o) + g$

Where $\Omega^2 = \frac{k}{M}$, $\Omega^2_\xi = \frac{k_\xi}{2M}$, $\Omega^2_\eta = \frac{k}{2M}$,
$H = \frac{C}{M}$, $\omega^2_\xi = \frac{k_s}{2m}$, $\omega^2_n = \frac{k_\eta}{2M}$,

Transformation of the Equations of Motion to the Main Coordinates

Let us introduce the following denotations

$\Omega_1{}^2 = \Omega^2_\xi + \Omega^2_n : \Omega^2{}_2 = \Omega^2_\xi - \Omega^2_\eta :$
$\omega_1{}^2 = \omega^2_\xi + \omega^2_\eta : \omega^2{}_2 = \omega^2_\xi - \omega^2_\eta : x = \alpha : \rho = \beta : \frac{H}{\varepsilon} = \frac{\mu H_1}{\varepsilon} :$

$a\cos\varphi_o = \mu P :$
$a\sin\varphi_o = \alpha Q : \mu G = g$ (11)
where; $\mu = \frac{\omega^2{}_2}{\omega^2{}_1} = \frac{\Omega^2{}_2}{\Omega^2{}_1} = \frac{k_\xi - k_n}{k_\xi + k_n}$ is the perturbation parameter.

After accounting for (11) in the equation system (10), it will assume the form

$\ddot{x}_1 = - x_1 \Omega^2 - x_1 \Omega^2{}_1(3 + \mu\cos 2\omega t) - \mu H_i \dot{x}_1 + x_2 \Omega^2{}_1(1 + \mu\cos 2\omega t) + x_3\Omega^2{}_1\mu\sin 2\omega t + \varepsilon(g - x_2 \Omega^2{}_1\mu\sin 2\omega t) + x_1\Omega^2{}_1\mu\sin 2\omega t + x_3 \Omega^2{}_1(1 - \mu\cos 2\omega t) \cdot [\mathrm{sgn}\,(v_o - \dot{x}_1) - x(v_o - \dot{x}_1) + \rho(v_o - \dot{x}_1)^3] :$
$\ddot{x}_2 = - x_1\omega^2{}_1(1 + \mu\cos 2\omega t) - x_2 \omega^2{}_1(1 + \mu\cos 2\omega t) + x_3\omega^2{}_1\mu\sin 2\omega t + \mu(P\sin\omega t + Q\cos\omega t) w^2$
$\ddot{x}_3 = - x_1\omega^2{}_1\mu\sin 2\omega t + x_2\omega^2{}_1\mu\sin 2\omega t - x_3\omega^2{}_1(1 - \mu\cos 2\omega t) + \mu(P\cos\omega t - Q\sin\omega t)w^2 + \mu G$ (12)

When introducing $\mu = \varepsilon = 0$ into the equation system (12), we obtain a homogeneous linear differential equation system
$\ddot{x}_1 + x_1(\Omega^2 + \Omega^2{}_1) - x_2\Omega^2{}_1 = 0$
$\ddot{x}_1 + \omega^2{}_1(x_2\ x_1) = 0$
$\ddot{x}_3 + \omega^2{}_1 x_3 = 0$ (13)

When assuming the solution of (13) in the form $x_i = A_i\cos\rho t$, $i = 1,2,3$, we find the following frequencies

$p^2{}_{1.2} = \frac{1}{2}[\ \Omega^2 + \Omega^2{}_1 + \omega^2{}_1 \pm\sqrt{\Omega^2 + \Omega^2{}_1 + \omega^2{}_1)^2 - 4\ \Omega^2\omega^2{}_1}]$ (14)
$p^2{}_3 = \omega^2{}_1$

Let us introduce the main coordinates ξ_i, for which at $\mu = \varepsilon = 0$ disjugation of the linear part of the first two equations of the system (12) will occur. Let us now multiply these equations by ξ_i and ξ_2, respectively, and add the sides. The result will be

$\ddot{x}_1\xi_1 + x_1 (\Omega^2 + \Omega^2{}_1)\xi_1 - x_2\Omega^2{}_1\xi_1 + \ddot{x}_2\xi_2 + \ddot{x}_1 w^2{}_1\xi_1 - \ddot{x}_1\omega^2{}_1\xi_1 = \mu[-x_1\Omega^2{}_1\cos 2\omega t\xi_1 - H_1\dot{x}_1\xi_1 + \xi_1 x_2\Omega^2{}_1\cos 2\omega t - \xi_1 x_3\Omega^2{}_1\sin 2\omega t + \xi_2 x_1\omega^2{}_1\cos 2\omega t - \xi_2 x_2 w^2{}_1\cos 2\omega t + \xi_2 x_3\omega^2{}_1\sin 2\omega t + \omega^2\xi_2(P\sin\omega t + Q\cos\omega t)] + \varepsilon\,\xi_1[g - x_2\Omega^2{}_1\mu\sin 2\omega t + x_3\Omega^2{}_1(1 - \mu\cos 2\omega t) + x_1\Omega^2{}_1\mu\sin 2\omega t]\cdot[\mathrm{sgn}(v_o - \dot{x}_1) - x(v_o - \dot{x}_1) + \rho(v_o - \dot{x}_1)^3]$ (15)

By denoting

$(\Omega^2 + \Omega^2{}_1)\xi_1 - w^2{}_1 = \xi_1\theta^2 - \Omega^2{}_1\xi_1 + \omega^2{}_1\xi_2 = \xi_2\theta^2$ (16)

We find

$(\Omega^2 + \Omega^2{}_1 - \theta^2)\xi_1 + \omega^2{}_1\xi_2 = 0$

$-\Omega^2{}_1\xi_1 + (\omega^2{}_1 - \theta^2)\xi_2 = 0$ (17)

In order for Equation (17) to be fulfilled for ξ_1 and ξ_2 different from zero, the following dependence must occur.

$$\begin{vmatrix} \Omega^2 + \Omega^2{}_1 - \theta^2 & -\omega^2{}_1 \\ -\Omega^2{}_1 & \omega^2{}_1 - \theta^2 \end{vmatrix} = 0$$

hence

$\Theta_1{}^2 = \rho_1{}^2$ and $\Theta_2{}^2 = \rho_2{}^2$ (18)

Let $\xi_1 = \xi'_1$ and $\xi_2 = \xi'_2$ be denoted for $\theta_1 = \rho_i$ · from the second equation of the system (16) we find

$\xi'_2 = y_1\,\xi'_1$ (19)

Where

$$\gamma_1 = \frac{\Omega^2{}_1}{\omega^2{}_1 - \rho^2{}_1}$$

Making use of the dependences (16) and (19), the Equation (15) is transformed to the form

$\ddot{x}_1 + x_1\rho_1{}^2 + \ddot{x}_2\gamma_1 + x_2\rho_1{}^2\gamma_1 = \mu[x_1\rho_1{}^2\gamma_1\cos 2\omega t - H_1\dot{x}_1 - \rho_1{}^2\gamma_1 x_2\cos 2\omega t + \rho_1{}^2\gamma_1 x_2\sin 2\omega t + \gamma_1\omega^2(P\sin\omega t + Q\cos\omega t)] + \varepsilon[g - x_2\Omega_1{}^2\mu\sin 2\omega t + x_3\Omega_1{}^2(1 - \mu\cos 2\omega t) + x_1\Omega_1{}^2\mu\sin 2\omega t]\cdot[\mathrm{sgn}(v_o - \dot{x}_1) - x(v_o - \dot{x}_1) + \beta(v_o - \dot{x}_1)^3]$ (20)

Analogously, for $\theta_2 = P_2$ the following are denoted; $\xi_1 = \xi''_1$ and $\xi''_2 = \xi''_2$ while

$\xi''_2 = \gamma_2\xi''_1$ (21)

where;

$$\gamma_2 = \frac{\Omega_1{}^2}{\omega_1{}^2 - \rho_2{}^2}$$

Taking (16) and (21) into account in (15), the equation will assume the form

$\ddot{x}_1 + x_1\rho_2{}^2 + \ddot{x}_2\gamma_2 + x_2\rho_2{}^2\gamma_2 = \mu[x_1\gamma_1\rho_2{}^2\cos 2\omega t - H_1\dot{x}_1 - x_2\gamma_2\rho_2{}^2\cos 2\omega t + [x_3\gamma_2\rho_2{}^2\sin 2\omega t + \gamma_2\omega^2(P\sin\omega t + Q\cos\omega t)] + \varepsilon[g - x_2\Omega_1{}^2\mu\sin 2\omega t + x_3\Omega_1{}^2(1 - \mu\cos 2\omega t) + x_1\Omega_1{}^2\mu\sin 2\omega t]\cdot[\mathrm{sgn}(v_o - \dot{x}_1) - x(v_o - \dot{x}_1) + \beta(v_o - \dot{x}_1)^3]$ (22)

Let us denote

$y_1 = \times_1 + \gamma_1\times_2$
$y_2 = \times_1 + \gamma_2\times_2$ (23)

The reserve dependences can be determined from the Equation (23)

$\times_1 = \beta_1 y_1 - \beta_2 y_2$ (24)
$\times_2 = \varphi(y_1 - y_2)$

Where; $\beta_1 = \frac{\gamma_2}{\gamma_2 - \gamma_1}$, $\beta_2 = \frac{\gamma_2}{\gamma_2 - \gamma_1}$, $\varphi_1 = \frac{1}{\gamma_2 - \gamma_1}$

Let us additionally assume that $\times_3 = y_3$

Taking (23) and (24) into account in (22), (20) and (12) we shall obtain the following differential equation system

$\ddot{y}_1 + \rho_1^2 y_1 = \mu[\rho_1^2 \gamma_1(\beta_1 y_1 - \beta_2 y_2)\cos2\omega t - H_1(\beta_1\dot{y}_1 - \beta_2\dot{y}_2) - \rho_1^2 \gamma_1\Psi\cdot(y_1 - y_2)\cos2\omega t + \rho_1^2 y_1 y_3\sin2\omega t + \omega^2\gamma_1(P\sin\omega t + Q\cos\omega t)] + \varepsilon[g - \mu\,\Omega_1^2)\Psi(y_1 - y_2)\cos2\omega t + \mu\,\Omega_1^2(\beta_2 y_1 - \beta_2 y_2)\sin2\omega t + \Omega_1^2 y_3(g - \mu\cos2\omega t)][\mathrm{sgn}(v_o - \beta_2\dot{y}_1 + \beta_2\dot{y}_2) - x(v_o - \beta_2\dot{y}_1 + \beta_2\dot{y}_2) + \rho(v_o - \beta_2\dot{y}_1 + \beta_2\dot{y}_2)^3]$: $\ddot{y}_2 + \rho_2^2 y_2 = \mu[\rho_2^2 \gamma_2(\beta_1 y_1 - \beta_2 y_2)\cos2\omega t - H_1(\beta_1\dot{y}_1 - \beta_2\dot{y}_2) + \rho_2^2\gamma_2\Psi(y_1 - y_2)\cos2\omega t + \rho_2^2 y_2 y_3\sin2\omega t + \omega^2\gamma_2(P\sin\omega t + Q\cos\omega t)] + \varepsilon[g - \mu\,\Omega_1^2)\Psi(y_1 - y_2)\sin2\omega t + \mu\Omega_1^2(\beta_1 y_1 - \beta_2 y_2)\sin2\omega t + \Omega_1^2 y_3(1 - \mu\cos2\omega t)][\mathrm{sgn}(v_o - \beta_2\dot{y}_1 + \beta_2\dot{y}_2) - x(v_o - \beta_2\dot{y}_1 + \beta_2\dot{y}_2) + \rho(v_o - \beta_1\dot{y}_1 + \beta_2\dot{y}_2)^3]$ (25)

$\ddot{y}_3 + \rho_3^2 y_3 = \mu[-\rho_3^2(\beta_1 y_1 - \beta_2 y_2)\sin2\omega t + \rho_3^2\Psi(y_1 - y_2)\sin2\omega t + \rho_3^2 y_3\cos2\omega t + \omega^2(P\cos\omega t + Q\sin\omega t) + G]$

After introducing the dimensionless time τ = ωt, we obtain

$\ddot{y}_1 + \lambda_1^2 y_1 = \mu[\lambda_1^2\gamma_1(\varepsilon_1 y_1 - \varepsilon_2 y_2)\cos2\tau - \lambda_1\hat{H}_1(\beta_1 y'_1 - \beta_2 y'_2) + \lambda_1^2 y_3\sin2\tau + \gamma_1(P\sin\tau + Q\cos\tau)] + \varepsilon[g + \mu\Omega_1^2(\varepsilon_1 y_1 - \varepsilon_2 y_2)\sin2\tau + \Omega_1^2 y_3(1 - \mu\cos2\tau)]\cdot \mathrm{sgn}\left(v_o - \frac{\lambda_1^2}{p_1^2}\right)(\beta_1\omega y'_1 - \beta_2\omega y'_2) + -\lambda_1\ddot{\times}_1(\lambda_1 v'_o - \beta_1 y'_1 + \beta_2 y'_2) + \omega\rho(\lambda_1 v'_o - \beta_1 y'_1 + \beta_2 y'_2)^3]$ $\ddot{y}_2 + \lambda_2^2 y_2 = \mu[\lambda_2^2\gamma_2(\varepsilon_1 y_1 - \varepsilon_2 y_2)\cos2\tau - \lambda_2\hat{H}_2(\beta_1 y'_1 - \beta_2 y'_2) + \gamma_2\lambda_2^2 y_3\sin2\tau + \gamma_2(P\sin\tau + Q\cos\tau)] + \varepsilon[g + \mu\Omega_1^2(\varepsilon_1 y_1 - \varepsilon_2 y_2)\sin2\tau + \Omega_1^2 y_3(1 - \mu\cos2\tau)]\cdot \mathrm{sgn}\left(v_o - \frac{\lambda_2^2}{p_2^2}\right)(\beta_1\omega y'_1 - \beta_2\omega y'_2) + -\lambda_2\ddot{\times}_2(\lambda_2 v''_o - \beta_1 y'_1 + \beta_2 y'_2) + \omega\rho(\lambda_1 v''_o - \beta_1 y'_1 + \beta_2 y'_2)^3$ $\ddot{y}_3 + \lambda_3^3 y_3 = \mu[-\lambda_3^2(\varepsilon_1 y_1 - \varepsilon_2 y_2)\sin2\tau + \lambda_3^2 y_3\cos2\tau + P\cos\tau + Q\sin\tau + \lambda_3^2\breve{G}]$

Where;
$y_i = \frac{dy_i}{p}$; $\lambda_i^2 = \frac{p_i^2}{\omega^2}$, i = 1, 2, 3

$\varepsilon_k = \beta_k - \varphi$, $k = 1,2$

$\ddot{\times}_k = \frac{\times}{p_k}$

$\breve{G} = \frac{G}{p_2^2}$; $\hat{H}_1 = \frac{H_1}{p_1}$; $v'_o = \frac{v_o}{p_1}$; $v''_o = \frac{v_o}{p_2}$; $\hat{H}_1 = \frac{H_2}{p_2}$

Zones of instability of the first order

The procedure of solving the equation system (26) consists in assuming two perturbation parameters μ and ε connected with parameter excitation and friction, respectively.
The sought periodic solutions of y_i (τ) are presented in the form of a double power series:

$$y_i(\tau) = y_{0,0} + \mu y_{0,1} + \mu^2 y_{0,2} + \ldots$$
$$+ \varepsilon(\overset{(i)}{y}_{0,1} + \mu \overset{(i)}{y}_{1,1} + \mu^2 \overset{(i)}{y}_{1,2} + \ldots) + \qquad (27)$$

Where: $\overset{(i)}{y}_{k,1} k,1 = 0,1,2, \ldots$ must fulfill the condition of periodicity. Periodic solutions are only possible for certain values of the parameters λ_i presented in the form of analogous series:

$$\lambda_i = n^2 + \mu\alpha_{0,1} + \mu\alpha_{0,1} + \ldots + \varepsilon(\alpha_{1,0} + \mu_{\alpha 1,1} + \mu^2\alpha_{2,2} + \ldots) + \qquad (28)$$

Where: $\alpha_{k,1}$ k,1 = 0,1,2 … are the unknown coefficients, which are determined from the condition of periodicity, avoiding in the solution terms unrestrictedly growing in time. For the resonance of the first order n^2 = 1 we shall determine the parametric instability zones, for which the frequency of parameter modulation fulfills, consecutively, the dependences $\omega \cong p_1$, $\omega \cong p_2$, and $\omega \cong p_1$. In series (27) and (28) for $\omega \cong p_1$ and $\omega \cong p_2$ we shall limit our considerations to the first powers of the small parameters μ and ε. On the other hand, for $\omega \cong p_3$ we shall limit ourselves in the calculations to the second approximation. In all the three cases we shall assume that $\mathrm{sgn}(v_o - \beta_1\omega y'_1 - \beta_2\omega y'_2) = 1$

Let

$$\lambda_2^2 = \nu_{2,1}^2\lambda_1^2$$

$$\lambda_3^2 = \nu_{3,1}^2\lambda_1^2 \qquad (29)$$

where:

$$\nu_{2,1} = \frac{p_2}{p_1}$$

$$\nu_{3,1} = \frac{p_3}{p_1}$$

and let us assume that $\nu_{2,1}$ and $\nu_{3,1}$ are not integers. Let us first consider the case $\omega \cong p_1$ assuming that

$$\overset{(2)}{y}_{o.o}(\tau) = \overset{(3)}{y}_{o.o}(\tau) = 0 \qquad (30)$$

The assumption is accounted for by a weak conjugation of the Equation (26) and ε≪1 and μ≪1. For μ = ε = 0 we shall obtain a disjugate system of three linear differential equations. For the resonance coordinate y_1 , the magnitude of oscillation of the other two main coordinates should be of the order of the small parameters μ and ε.
Let us substitute the series (27) and (28) in the differential Equation (26) taking into consideration the dependences (29) and (30) and the expansion.

$$\lambda_1 \cong 1 + \mu\frac{\alpha 0,1}{2} + \varepsilon\frac{\alpha 0,1}{2} + \ldots \quad \tau \qquad (31)$$

After equation to zero the coefficients at the same powers ε and μ, we obtain a system of recurrent differential equations

$$y''_{o.o} + y_{o.o} = 0$$

$$\frac{g}{p^2}$$

$y''_{o.o} + y_{o.o} = - a_{1,0}y_{o.o} +$ - - $g\ddot{x}_1 v_o$ $+ g\ddot{x}_1\beta_1 y'_{o.o} + g\omega\rho(v_o)^3 - 3g\omega\rho(v_o)^2\beta_1 y'_{o.o} + 3g\omega\rho v_o\beta_1(y'_{o.o})^2 + g\omega\rho\beta_1(y'_{o.o})^3$; $y''_{o.o} + y'_{o.o} = - a_{0,1}y_{o.o} + \gamma_1\varepsilon_1 y_{o.o}$ $\cos2\tau - \hat{H}_1\beta_1 y'_{o.o} + \gamma_1 P\sin\tau + \gamma_1 Q\cos\tau$;$y''_{1,0} + v_{2,1}y_{1,0} = g \cdot v_{2,1}/P_2 g v_{2,1}$ $\ddot{x}_1 v''_o + g v_{2,1}$ $\ddot{x}_2\beta_1 y'_{o.o} + g\omega\rho v_{2,1}(v''_o)^3 + 3g\omega\rho v_{2,1}\beta_1(v''_o)^2 \beta_1 y'_{o.o} + 3g\omega\rho v_{2,1}v''_o\beta_1(y'_{o.o})^2 + g\omega\rho\beta_1(y'_{o.o})^3$: (1) (32)

$y''_{0,1} + v_{2,1}y_{0,1} = g \cdot \gamma_2 v_{2,1}\varepsilon_1 y'_{o.o}\cos2\tau - v_{2,1}\hat{H}_2\beta_1 y'_{o.o} + \gamma_2 P\sin\tau$

$\gamma_1 Q\cos\tau$; $y''_{1,0} + v_{3,1}y_{1,0} = 0$

$y''_{1,0} + v_{3,1}y_{1,0} = - v_{2,1}\varepsilon_1 y_{o.o}\sin2\tau + P\cos\tau - Q\sin\tau + v_{3,1}\breve{G}$

Assuming the solution of the first equation of the system (32) in the form

$$y_{0,0}^{(1)} = a_1\cos\tau + b_1\sin\tau \tag{33}$$

We obtain the following from the second equation

$$y''_{1,0} + y_{1,0} = \frac{g}{p_1^2} - g\ddot{x}_1 v_o + g\omega\rho(v'_o) + \frac{3}{2}g\omega\rho v'_o\beta_1(a_1 + b_1) + \cos\tau[-a_{1,0}a_1 + g\ddot{x}\beta_1 b_1 - 3g\omega\rho(\nu'_0)^2\beta_1 b_1 - \frac{3}{4}g\omega\rho\beta_1 b_1 + \frac{3}{4}g\omega\rho\beta_1 b_1 a_1] + \sin\tau[-a_{1,0}b_1 - g\ddot{x}\beta_1 a_1 + 3g\omega\rho(\nu'_0)^2\beta_1 a_1 + \frac{3}{4}g\omega\rho\beta_1 b_1 a_1 + \frac{3}{4}g\omega\rho\beta_1 a_1] + \frac{3}{4}g\omega\beta\nu'_o\beta_1(b_1 + a_1) + \cos2\tau + 3g\omega\rho\nu'_o\beta_1 a_1 b_1\sin2\tau + \cos3\tau[-\frac{1}{4}g\omega\rho\beta_1 b_1 + \frac{3}{4}\beta_1 b_1 a_1] + \sin3\tau(-\frac{1}{4}g\omega\rho\beta_1 a_1 + \frac{3}{4}g\omega\rho\beta_1 b_1 a_1) \tag{34}$$

From the condition of periodicity we obtain two algebraic equations

$$- a_{1,0}a_1 + (g\ddot{x}_1\beta_1 - 3g\omega\rho(\nu'_0)^2\beta_1 - \frac{3}{4}g\omega\rho\beta_1 A_1)b_1 = 0$$

$$- (g\ddot{x}_1\beta_1 - 3g\omega\rho(\nu'_0)^2\beta_1 - \frac{3}{4})g\omega\rho\beta_1 A_1)a_1 - a_{1,0}b_1 = 0 \tag{35}$$

Where

$$A_1^2 = a_1^2 + b_1^2$$

For the non-zero a_1 and b_1 the following relation must occur

$$\begin{vmatrix} - a_{1,0} & g\ddot{x}_1\beta_1 - 3g\omega\rho(\nu'_0)^2\beta_1 - \frac{3}{4}g\omega\rho\beta_1 A_1^3 \\ - (g\ddot{x}_1\beta_1 - 3g\omega\rho(\nu'_0)^2\beta_1 - \frac{3}{4}g\omega\rho\beta_1 A_1^3 & - a_{1,0} \end{vmatrix} = 0 \tag{36}$$

Thence

$$- a_{1,0}^2 \quad g\ddot{x}_1\beta_1 - 3g\omega\rho(\nu'_0)^2\beta_1 - \frac{3}{4}g\omega\rho\beta_1 A_1^3 \,{}^2 = 0 \tag{37}$$

The only real solution of (37) is

$a_{1,0} = 0$

$$A_1^2 = \frac{\ddot{x} - 3g\omega\rho(\nu'0)2}{\frac{3}{4}\omega\rho\beta1} \tag{38}$$

The following function is the solution is (34)

(1) $\frac{g}{p_1^2}$ 2 2 2 2 2

$y_{1,0} = \quad - g\ddot{x}_1 v_o + g\omega\rho(v'_o)^3 + \frac{3}{2} g\omega\rho v'_o \beta_1 A_1 + \frac{1}{2} g\omega\rho v'_o \beta_1 (b_1 + a_1) + \cos 2\tau + g\omega\rho v'_o \beta_1 a_1 b_1 \sin 2\tau + (\frac{1}{32} g\omega\rho\beta_1 b_1$

$+ \frac{3}{32} g\omega\rho\beta_1 b_1 a_1) \cdot \cos 3\tau + (\frac{1}{32} g\omega\rho\beta_1 a_1$ (39)

$+ \frac{3}{32} g\omega\rho\beta_1 b_1 a_1) \cdot \sin 3\tau$

The solution omits the general integral of the homogenous equation by associating it to $y_{o,o}$.
When substituting (33) in the fourth and sixth equation of the equation system (32), after transformations, we obtain

$y''_{1,0(2)} + v_{2,1}^2 y_{1,0(2)} = v_{2,1}^2 - v_{2,1} \frac{g}{p^2} g\ddot{x}_2 v_0''$

$+ g\omega\rho_{2,1} (v''_o)^3 + \frac{3}{2} v_{2,1} g\omega\rho v''_o \beta_1 A_1$

$+ \cos\tau\, [v_{2,1} g\ddot{x}_2 \beta_1 b_1 - 3 g\omega\rho v_{2,1}$

$(v''_o)^2 \beta_1 b_1 + \frac{3}{4} g\omega\rho\beta_1 A_1 b_1] + \sin\tau\, [-v_{2,1} g\ddot{x}_2 \beta_1 a_1$

$+ 3 g\omega\rho v_{2,1} (v''_o)^2 \beta_1 a_1$

$+ \frac{3}{4} g\omega\rho\beta_1 A_1 a_1] + \frac{3}{2} v_{2,1} g\omega\rho v''_0$

$\beta_1 (b_1 - a_1) \cos 2\tau + 3 v_{2,1} g\omega\rho v'' \beta_1 a_1 - b_1$

$\sin 2\tau + (-\frac{1}{4} g\omega\rho\beta_1 b_1 + \frac{3}{2} g\omega\rho\beta_1 b_1 a_1)$

$\cos 3\tau + (-\frac{1}{4} g\omega\rho\beta_1 a_1 + \frac{3}{2} g\omega\rho\beta_1 b_1 a_1) \sin 2\tau$ (40)

$y''_{1,0(3)} + v_{3,1}^2 y_{1,0(3)} = 0$ (41)

The following functions are the particular solutions of the above equations:

$y_{1,0(2)} = \frac{g}{p^2} g\ddot{x}_2 v''_0 + g\omega\rho v_{2,1} (v''_o)^3$

$+ \frac{3}{2 v_{2,1}} g\omega\rho v''_o \beta_1 A_1 + \frac{1}{v_{2,1}^2 - 1}$

$[v_{2,1} g\ddot{x}_2 \beta_1 - 3 g\omega\rho v_{2,1} (v''_o)^2 \beta_1$

$- \frac{3}{4} g\omega\rho\beta_1 A_1] b_1 \cos\tau + \frac{1}{v_{2,1}^2 - 1}$

$[- v_{2,1} g\ddot{x}_2 \beta_1 a_1 + 3 g\omega\rho v_{2,1} (v''_o)^2$

$\beta_1 a_1 + \frac{3}{4} g\omega\rho\beta_1 A_1 a_1] \sin\tau + \frac{3 v_{2,1}}{2(v_{2,1}^2 - 4)}$

$g\omega\rho v''_0\beta_1 a_1 b_1 \sin 2\tau + \frac{1}{v_{2,1}^2 - g}$

$(-\frac{1}{4} g\omega\rho\beta_1 a_1^3 + \frac{3}{4} g\omega\rho\beta_1 b_1 a_1^2)\cos 3\tau$ (42)

$\frac{1}{v_{2,1}^2 - g}(-\frac{1}{4} g\omega\rho\beta_1 a_1^3 + \frac{3}{4} g\omega\rho\beta_1 b_1 a_1^2)\sin 3\tau$

And

$y_{1,0}^{3} = 0$ (43)

By means of substituting (39) in the third equation of the system (32), we shall obtain

$y''^{(1)}_{0,1} + y^{(1)}_{0,1} = (a_{0,1}a_1 + \frac{1}{2}\gamma_1\varepsilon_1 a_1 - \hat{H}_1\beta_1 b_1 + \gamma_1 Q)$

$\cos\tau + (a_{0,1}b_1 + \frac{1}{2}\gamma_1\varepsilon_1 b_1 + \hat{H}_1\beta_1 a_1 + \gamma_1 P)$ (44)

$\sin\tau + \frac{1}{2}\gamma_1\varepsilon_1 a_1 \cos 3\tau + \frac{1}{2}\gamma_1\varepsilon_1 a_1 \sin 3\tau$

we shall avoid terms unrestrictedly growing in time in its solution if the following equations are fulfilled.

$(a_{0,1} - \frac{1}{2}\gamma_1\varepsilon_1)a_1 + \hat{H}_1\beta_1 b_1 = \gamma_1 Q)$

$-\hat{H}_1\beta_1 a_1 + (a_{0,1} - \frac{1}{2}\gamma_1\varepsilon_1)b_1 = \gamma_1 P$ (45)

For the case of $P = Q$, after transformations, we obtain the following from (45)

$$a_{0,1} = \sqrt{\frac{P}{A}\gamma_1^2 + \frac{1}{4}\gamma_1^2\varepsilon_1^2 - \hat{H}_1^2\beta_1^2 \pm \sqrt{\left(\frac{P}{A}\gamma_1^2\right)^2 + \gamma_1^3\varepsilon_1\frac{P}{A} \pm 2\hat{H}_1\beta_1\frac{P}{A}\gamma_1^2\varepsilon_1}}$$

The particular solution of (44) is

$y^{(1)}_{0,1} = -\frac{1}{16}\gamma_1\varepsilon_1(a_1\cos 3\tau + b_1\sin 3\tau)$ (47)

Taking (33) into consideration in the fifth and seventh equation of the system (32) we find the particular solutions

$y_{0,1} = \frac{1}{v_{2,1}^2 - 1}(\frac{1}{2}v_{2,1}\gamma_1\varepsilon_1 a_1 - v_{2,1}\hat{H}_2\beta_1 b_1 + \gamma_2 Q)\cos\tau$

$+ \frac{1}{v_{2,1}^2 - 1}(\frac{1}{2}v_{2,1}\gamma_1\varepsilon_1 b_1 + v_{2,1}\hat{H}_2\beta_1 a_1 + \gamma_2 P)\sin\tau + \frac{v_{2,1}^2\gamma_2\varepsilon_1}{2(v_{2,1}^2 - 9)}$

$a_1\cos 3\tau + \frac{v_{2,1}^2\gamma_2\varepsilon_1}{2(v_{2,1}^2 - 9)} b_1\sin 3\tau$

$y^{(3)}_{0,1} = \breve{G} + \frac{1}{v_{3,1}^2 - 1}(-\frac{1}{2}v_{3,1}\varepsilon_1 b_1^2 + P)\cos\tau$

$+ \frac{1}{v_{3,1}^2 - 1}(\frac{1}{2}v_{3,1}\varepsilon_1^2 a_1 + Q)\sin\tau + \frac{v\ \varepsilon}{2(v\ \ -9)}$ (49)

$b_1\cos3\tau + \frac{v\ \varepsilon_{3,1}^{2}}{2(v_{3,1}^2 - 9)}a_1\sin3\tau$

We have thus determined the particular terms of the series (27) and (28), limiting the calculations to the first approximation.

Let us now concentrate on the analysis of the case of $\omega \cong p_2$. The solutions will be sought, as has been done previously, in the form of the series (27) and (28) for $i = 2$. From (28) we obtain

$\lambda_2 = 1 + \varepsilon\frac{a_{1,0}}{a} + \mu\frac{a}{2} + \dots$ (50)

Let us denote that

$\lambda_{1}^{2} = v_{1,2}^{2}\,\lambda_{2}^{2}$

$\lambda_{3}^{2} = v_{3,2}^{2}\,\lambda_{2}^{2}$

where $v_{1,2} = \frac{p_1}{p_2}$ (51)

$v_{3,2} = \frac{p_3}{p_2}$

and $v_{1,2}$ and $v_{3,2}$ are not integers.

Analogously to (30), we have

$y_{0,0}^{(1)}(\tau) = y_{0,0}^{(3)} = 0$ (52)

Substituting (27) in (26), with (50, (51) and (52) taken into account, after equating to zero the coefficients at the same powers of μ and ε, we obtain

$y''^{(1)}_{1,0} + v_{1,2}^2 y^{(1)}_{1,0} = \frac{g}{p} - g\ddot{x}_2 v'_0 - g\ddot{x}_2 v_{1,2}$

$\beta_2 y'^{(2)}_{0,0} + g\omega\rho v_{2,1}(v'_0)^3 + 3g\omega\rho\beta_2 v_{2,1}^2$

$(v'_0)^2 y'^{(2)}_{0,0} + 3g\omega\rho\beta_2 v'_0(y'^{(2)}_{0,0})^2 + g\omega\rho\beta_2(y'^{(2)}_{0,0})^3$

$y''^{(1)}_{1,0} + v_{1,2}^2 y^{(1)}_{1,0} = -\gamma_1 v_{1,2}\varepsilon y^{(2)}_{0,0}\cos2\tau + v_{1,2}$

$\hat{H}_1\beta_2 y'^{(2)}_{0,0} + \beta_1 P\sin\tau + \gamma_1 Q\cos\tau$;

$y''^{(2)}_{1,0} + y^{(2)}_{1,0} = 0$;

$y''^{(2)}_{1,0} + y^{(2)}_{1,0} = -\alpha_{1,0}y^{(2)}_{0,0} + \frac{g}{p} - g\ddot{x}_2 v''_o$

$- g\ddot{x}_2\beta_2 y''^{(2)}_{o,o} + g\omega\rho(v''_o)^3 + 3g\omega\rho(v''_o)^2\beta_2 y'^{(2)}_{o,o}$

$+3g\omega\rho v''_o\beta_2(y'^{(2)}_{o,o})^2 + g\omega\rho\beta_2(y'^{(2)}_{o,o})^3$; (53)

$y''^{(2)}_{1,0} + y^{(2)}_{1,0} = -\alpha_{1,0}y^{(2)}_{0,0} + \gamma_2\varepsilon_2\, y^{(2)}_{0,0}\cos2\tau$

$+ \hat{H}_2\beta_2 y'_{0,0} + \gamma_2 P\sin\tau + \gamma_2 Q\cos\tau$

$y''_{1,0} + v^2_{3,2} y_{1,0} = 0;$

$y''_{1,0} + v_{3,2} y_{1,0} = v_{3,}\varepsilon_2 y_{0,0}\sin 2\tau$

$+ P\cos\tau - Q\sin\tau + v_{3,2}\breve{G}$ 2

After substituting the following in the fourth equation of the system (53)

$$y^{(2)}_{o,o} = a_2\cos\tau + b_2\sin\tau \quad (54)$$

and using the trigonometric relations, we obtain

$y''^{(2)}_{1,0} + y^{(2)}_{1,0} = \frac{g}{p} - g\ddot{x}_2 v''_o + g\omega\rho(v''_o)^3$ 2 2

$+ \frac{3}{2} g\omega\rho(v''_o)_2 (a^2_2 + b^2_2) + (a_{1,0}a_2 - g\ddot{x}_2\beta_2 b_2$ 3 3

$+ 3g\omega\rho(v''_o)^2\beta_1 b_1 + \frac{3}{4} g\omega\rho\beta_2 b_2$

$+ \frac{3}{4} g\omega\rho\beta_2 b^3_2 a_2)\cos^3\tau + (- a_{1,0}b_2$

$+ g\ddot{x}_2\beta_2 a_2 - 3g\omega\rho(v''_o)^2\beta_2 a_2$

$+ \frac{3}{4} g\omega\rho\beta_2 b^3_2 a^2_2 - \frac{3}{4} g\omega\rho\beta_2 a_2)\sin^3\tau^2$

$+ \frac{3}{2} g\omega\rho v''_o\beta_2(b^2_2 + a^2_2)\cos 2\tau$

$+ 3g\omega\rho v''_o\beta_2 a_2 b^2_2\sin_2\tau + \frac{1}{4}$

$$+ \frac{1}{4}(a_2 + 3b_2)^2 g\omega\rho\beta_2 a_2\sin^3 3\tau \quad (55)$$

From the condition of periodicity of the solution we get

$- a_{1,0}a_2 + (- g\ddot{x}_2\beta_2 + 3g\omega\rho(v''_o)\beta_2 + 3g\omega\rho\beta_2 A_2)b_2 = 0$ 3 2

$(g\ddot{x}_2\beta_2 - 3g\omega\rho(v''_o)^2\beta_2 - \frac{3}{4} g\omega\rho\beta_2 A_2)a_2$ 3 2

$$- a_{1,0}b_2 = 0 \quad (56)$$

Where $A^2_2 = a_2 = ^2 b_2$ 2

For the non – zero a_2 and b_2 the main determinant of the equation system (56) must equal zero. From this condition we obtain

$a_{1,0} = 0$

2

$$A_2^2 = \frac{\ddot{x}_2 - 3g\omega\rho(v''_o)^2}{\frac{3}{4}\omega p\beta_2^2} \quad (57)$$

The particular solution of the Eq. (55) is

$$y_{1,0}^{(2)} = \frac{g}{p} - g\ddot{x}_2 v''_o + g\omega\rho(v''_o)^3 + \frac{3}{2} g\omega\rho v''_o \beta_2^2 A_2^2$$

$$+ \frac{3}{2} g\omega\rho v''_o \beta_2^2 (b_2^2 + a_2^2) \cdot \cos 2\tau$$

$$+ g\omega\rho v''_o \beta_2^2 a_2 b_2 \sin 2\tau + \frac{1}{32}(3a_2^2 + b_2^2) g\omega\rho\beta_2^3 b_2$$

$$\mathrm{Cos}3\tau + \frac{1}{32}(3b_2^2 + a_2^2) g\omega\rho\beta_2 a_2 \sin 3\tau \quad (58)$$

Making use of (54) in the first and sixth equation of the system (53) we obtain their particular integrals

$$y_{1,0}^{(2)} = \frac{g}{p} - g\ddot{x}_2 v''_o - g\omega\rho v'_{1,2}(v'_o)^3$$

$$+ \frac{3}{2v_{1,2}} g\omega\rho v'_o \beta_2^2 A_2^2 + \frac{1}{v_{1,2}^2 - 1}$$

$$-(-g\ddot{x}_1 v_{1,2}\beta_2 - 3g\omega\rho\beta_2 v^2_{1,2}(v'_o)^2$$

$$+ \frac{3}{2} g\omega\rho\beta_2^3 A_2^2) b_2 \cos\tau + \frac{1}{v_{1,2}^2 - 1}$$

$$(gx_1 v_{1,2}\beta_2 - 3g\omega\rho\beta_2 v^2_{1,2}(v'_o)^2$$

$$+ \frac{3}{4} g\omega\rho\beta_2^3 A_2^2) a_2 \sin\tau + \frac{3v_{1,2}}{2(v_{1,2}^2 - 1)} g\omega\rho\beta_2^2$$

$$v'_o(b_2^2 + a_2^2)\cos\tau + \frac{3v_{1,2}}{2(v_{1,2}^2 - 4)} g\omega\rho\beta_2^2 v'_o a_2 b_2$$

$$\sin 2\tau \cdot \frac{1}{4}(b_2^2 + 3a_2^2) \frac{g\omega\rho\beta_2^3}{2(v_{1,2}^2 - 9)}$$

$$b_2 \cos 3\tau + \frac{1}{4}(a_2^2 + 3b_2^2) \frac{g\omega\rho\beta_2^3}{2(v_{1,2}^2 - 9)} \sin 3\tau \quad (59)$$

and

$$y_{1,0}^{(3)} = 0 \quad (60)$$

The substitution of (54) in the fifth equation of the system (53) gives

$$y_{0,1} + y_{0,1} = (-a_{0,1}a_2 - \frac{1}{2}\gamma_2\varepsilon_2 a_2 + \hat{H}_2\beta_2 b_2 + \gamma_2 Q)$$

$$\cos\tau + \left(-a_{0,1}b_2 - \frac{1}{2}\gamma_2\varepsilon_2 a_2 + \hat{H}_2\beta_2 a_2 + \gamma_2 P\right) \tag{61}$$

$$\sin\tau + \frac{1}{2}\gamma_2\varepsilon_2 a_2\cos3\tau - \frac{1}{2}\gamma_2\varepsilon_2 b_2\sin3\tau$$

The following is obtained from the condition of periodicity after transformations and after assuming that $P = Q$.

$$a_{0,1}^4 + 2a_{0,1}^2\left(\hat{H}_2\beta_2 - \frac{1}{4}\gamma_2^2\varepsilon_2^2 - \frac{P}{A}\gamma_2^2\right)$$

$$+ \left(\hat{H}_2\beta_2 - \frac{1}{4}\gamma_2^2\varepsilon_2^2\right)^2 + \frac{1}{2}\gamma_2^4\varepsilon_2^2\frac{P}{A}$$

$$- 2\gamma_2^2\beta_2\hat{H}_2\frac{P}{A}\left(\hat{H}_2\beta_2 - \frac{1}{4}\gamma_2^2\varepsilon_2^2\right)^2 = 0 \tag{62}$$

Thence

$$a_{0,1} = \sqrt{\frac{P}{A}\gamma_2^2 + \frac{1}{4}\gamma_2^2\varepsilon_2^2 - \hat{H}_2\beta_2 \pm \sqrt{\left(\frac{P}{A}\gamma_2^2\right)^2 + \gamma_2^4\varepsilon_2^4\frac{P}{A} - 2\hat{H}_2\beta_2\frac{P}{A}\gamma_2^3\varepsilon_2^2}} \tag{63}$$

$$y_{0,1}^{(2)} = \gamma_2\varepsilon_2(a_2\cos3\tau + b_2\sin3\tau) \tag{64}$$

on the other hand, after substituting (54) in the second and seventh equation of (53), we shall find the particular solutions

$$y_{0,1}^{(1)} = \frac{1}{v_{2,1}^2 - 1}\left(\frac{1}{2}v_{2,1}\gamma_1\varepsilon_2 a_2 - v_{2,1}\hat{H}_1\beta_2 b_2 + \gamma_1 Q\right)\cos\tau$$

$$+ \frac{1}{v_{2,1}^2 - 1}\left(\frac{1}{2}v_{2,1}\gamma_1\varepsilon_2 b_2 - v_{2,1}\hat{H}_1\beta_2 a_2 + \gamma_1 P\right)\sin\tau + \frac{v_{2,1}^2\gamma_1\varepsilon_2}{2(v_{2,1}^2 - 9)}$$

$$a_1\cos3\tau - \frac{v_{1,2}^2\gamma\varepsilon_2}{2(v_{1,2}^2 - 9)}b_2\sin3\tau \tag{65}$$

$$y_{0,1}^{(3)} = \breve{G} + \frac{1}{v_{3,2}^2 - 1}\left(\frac{1}{2}v_{3,2}\varepsilon_2 b_2 + P\right)\cos\tau$$

$$+ \frac{1}{v_{3,1}^2 - 1}\left(\frac{1}{2}v_{3,2}\varepsilon_2 a_2 + Q\right)\sin\tau + \frac{v_{3,2}^2\varepsilon_2}{2(v_{3,2}^2 - 9)} \tag{66}$$

$$b_1\cos3\tau - \frac{v_{3,2}^2\varepsilon_2}{2(v_{3,2}^2 - 9)}a_2\sin3\tau$$

Finally, let us consider the case of $\omega \cong p_3$. Periodic solutions are possible for particular value of the parameter λ_3.

$$\lambda_3 \cong 1 + \varepsilon\frac{a_2^0}{2} + \mu\frac{a_2^{1,0}}{2} + \varepsilon^2\frac{a_2}{2} \tag{67}$$

Let us denote that

$\lambda_1^2 = v_{1,3}^2\lambda_3^2$

$\lambda_2^2 = v_{2,3}^2\lambda_3^2$

where;

$$v_{1,2} = \frac{p}{p} \qquad (68)$$

$$v_{3,2} = \frac{p}{p_3}$$

and $v_{1,3}$ and $v_{2,3}$ on assumption are not integers. Similarly to the previous considerations, assuming that

$$y^{(1)}_{0,0}(\tau) = y^{(1)}_{0,0}(\tau) = 0 \qquad (69)$$

we obtain the following recurrent differential equation system from the equation system (26)

$$y''^{(1)}_{1,0} + v^2_{1,3}\, y^{(1)}_{1,0} = (g + Q_1 y^{(1)}_{1,0})^2 \left[\frac{v}{p} - v^2_{1,3}\, \ddot{x}_1 v'_o + \omega\rho v^2_{1,3} (v'_o)^3\right]$$

$$y''^{(1)}_{1,0} + v^2_{1,3} y^{(1)}_{1,0} = \gamma_1 v_{1,3}\, y''^{(3)}_{o,o} \sin 2\tau + \gamma_1 (P\sin\tau + Q\cos\tau)$$

$$y''^{(1)}_{2,0} + v^2_{1,3} y^{(1)}_{1,0} = -v_{1,3} a^2_{1,0} y^{(1)}_{1,0} + (g + \Omega_1 y_{0,0})^{2\,(3)}$$

$$\left[\frac{a_{1,0}}{2} - v^2_{1,3}\, \ddot{x}_1 a_{1,0} v'_o + \nu_{1,3}\ddot{x}_1(-\beta_1 y'_{1,0} + \beta_2 y'^{(1)}_{1,0} + \beta_2 y'^{(2)}_{1,0} + \omega\rho\nu^2_{1,3})^{(2)}_3\right]$$

$$(\nu'_o)^2\left(\frac{a_{1,0}}{2} - v^2_{1,3}\, v'_o + \beta_1 y'_{1,0})\right]^{(2)}$$

$$+ \Omega^2_1 y^{(3)}_{1,0}) \left[\frac{1}{p_3} - v^2_{1,3}\, \ddot{x}_1 v'_o + \omega\rho\nu^2_{1,3} (v'_o)^3\right]$$

$$y''^{(1)}_{2,0} + v^2_{1,3} y^{(1)}_{1,0} = -v_{1,3} a^2_{0,1} y_{1,0} + v^{(1)}_{1,3}\gamma_1(\varepsilon_1 y^2_{0,1} - \varepsilon_2 y^{(1)}_{0,1})\cos 2\tau^{(2)}$$

$$+ v_{1,3}\hat{H}_1(\beta_1 y'^{(1)}_{0,1} - \beta_2 y'^{(2)}_{0,1} + v_{1,3}\gamma_1(a^2_{0,1} y_{0,0} + y^{(3)}_{0,1})\sin 2\tau^{(3)}$$

$$y''^{(1)}_{1,1} + v^2_{1,3} y^{(1)}_{1,1} = -v_{1,3}(a^2_{0,1} y_{1,0} + a_{1,0} y^{(1)}_{0,1})^{(1)}$$

$$+ v^2_{1,3}\gamma_1(\varepsilon_1 y^{(1)}_{1,0} - \varepsilon_2 y^{(1)}_{1,0})^{(2)} \cos 2\tau$$

$$- v_{1,3}\hat{H}_1(\beta_1 y^{(2)}_{1,0} - \beta_2 y^2_{1,0}) + v_{1,3}\gamma_1(a_{1,0} y^{(3)}_{0,0} + y^{(3)}_{0,0})\sin 2\tau$$

$$+ (g + \Omega^2_1 y^{(3)}_{0,0}) \left[\frac{a_{0,1}}{p_3} - v^2\, \ddot{x}_1 v'_o + \nu_{1,3}\ddot{x}_1(-\beta_1 y'_{0,1} + \beta_2 y'^{(1)}_{0,1}) + \omega\rho\nu^2_{1,3})^{(2)}\right.$$

$$(\nu'_o)^2(\nu^2_{1,3} v'_o \frac{a_{0}}{2} - \beta_1 y'_{0,1} + \beta_2 y'^{(1)}_{0,1})^{(2)}$$

$$+ \Omega^2_1(y_{0,1} + y_{0,0}\cos 2\tau)$$

$$\left(\frac{1}{p_3} - v^2_{1,3}\, \ddot{x}_1 v'_o + \omega\rho\nu^3_{1,3} (v'_o)^3\right); \qquad (70)$$

$$y''^{(2)}_{1,0} + v^2_{1,3}\, y^{(2)}_{1,0} = (g + \Omega_1 y_{0,0})^{2\,(3)} \left[\frac{v}{p} - v^2_{2,3}\, \ddot{x}''_o + \omega\rho\nu^2_{2,3} (v''_o)^3\right];$$

$$y''^{(2)}_{0,1} + v^2_{2,3}\, y^{(2)}_{1,0} = v_{2,3}\gamma_2 y''^{(3)}_{o,o}\sin 2\tau + \gamma_2(P\sin\tau + Q\cos\tau);$$

(2) 2 (2) 2 (2) 2 (3)

$y''_{2,0} + v_{2,3}\, y_{1,0} = -\, v_{2,3} a_{1,0} y_{1,0} + (g + \Omega_1 y_{0,0})$

$[\frac{a_{1,0}}{p^{\frac{2}{3}}} - v_{1,3}^{2} \ddot{x}_2 v'_0 a_{1,0} + \nu_{1,2} \ddot{x}_1 (\frac{2}{} \beta_1 y'^{(1)}_{1,0} + \beta_2 y'^{(2)}_{1,0} + \omega\rho\nu^2_{2,3} (\nu''_o)^3$

$(\frac{a_{1,0}}{2} v_{1,3}\, v''_o + -\, \beta_1 y'^{(1)}_{1,0} + \beta_1 y'^{(2)}_{1,0})]$

$+ \Omega_1^2 y^{(3)}_{1,0} [\frac{1}{p^{\frac{2}{3}}} - v_{1,3}\, \ddot{x}_1 v''_o + \omega\rho\nu_{1,3} (\nu''_o)^3]$

$y''^{(2)}_{0,2} + v_{2,3}^2 y^{(2)}_{0,2} = -\, v_{2,3} a_{0,1} y_{1,0} + v^{(2)}_{2,3} \gamma_2 (\varepsilon_1 y^{(1)}_{0,1} - \varepsilon_2 y^{(2)}_{0,1}) \cos 2\tau$
$+ v_{1,3} \hat{H}_2 (\beta_1 y'_{1,0} - \beta_2 y'_{0,1} + v_{2,3} \gamma_1 (a_{0,1} y_{0,0} + y_{0,1}) \sin 2\tau$

$y''_{1,1} + v_{2,3}\, y_{1,1} = -\, v_{2,3} (a_{0,1} y_{1,0} + a_{1,0} y_{0,1}$

$+ v_{2,3}^2 \gamma_2 (\varepsilon_1 y^{(1)}_{1,0} - \varepsilon_2 y^{(2)}_{1,0}) \cos 2\tau$

$-\, v_{2,3} \hat{H}_2 (\beta_1 y^{(1)}_{1,0} - \beta_2 y^{(2)}_{1,0}) + v_{2,3}^2 \gamma_2 (a_{1,0} y^{(3)}_{0,0} + y^{(3)}_{0,0}) \sin 2\tau$

$(g + \Omega_1 y^{2}_{0,0})_{1,3} \cdot [\frac{a_{1,0}}{p^{\frac{2}{3}}}$

$-\, v_{2,3}^{2} \ddot{x}_2 a_{1,0} v'_0 - \nu_{2,3} \ddot{x}_2 (\nu''_o)^2$

$(v_{2,3}\, v''_o \frac{a_{0,1}}{2} - \beta_1 y'_{0,1} + \beta_2 y'_{0,0})]$

$+ \Omega_1^2 (y^{(3)}_{0,1} + -\, y^{(3)}_{0,0} \cos 2\tau)$

$[\frac{1}{p^{\frac{2}{3}}} - v_{2,3}\, \ddot{x}_2 v''_o + \omega\rho\nu_{2,3} (\nu''_o)^3]$

$y''^{(3)}_{0,0} + y^{(3)}_{0,0} = 0$

$y''^{(3)}_{1,3} + y^{(3)}_{1,0} = -\, a_{0,0} y^{(3)}_{0,0}$

$\cos 2\tau + P \cos\tau - Q \sin\tau + \breve{G}$

$y''^{(3)}_{2,0} + y^{(3)}_{2,0} = -\, a_{1,0} y_{1,0} - a^{(3)}_{2,0} y^{(3)}_{0,0}$

$y''^{(3)}_{0,2} + y^{(3)}_{0,2} = -\, a_{0,1} y_{0,1} - a^{(3)}_{0,2} y^{(3)}_{0,0}$

$-\, \varepsilon_1 y^{(1)}_{0,1} \sin 2\tau + \varepsilon_2 y^{(2)}_{0,1} \sin 2\tau + a_{0,1} y^{(3)}_{0,0} \cos 2\tau$

$+ y^{(3)}_{0,1} \cos 2\tau + a_{0,1} \breve{G}$

$y''^{(3)}_{1,1} + y^{(3)}_{1,1} = -\, a_{0,1} s_{0,0} - a^{(3)}_{0,1} y^{(3)}_{0,1}$

$a_{0,1} y^{(3)}_{1,0} - \varepsilon_1 y^{(3)}_{1,0} - \varepsilon_1 y^{(1)}_{1,0} \sin 2\tau$

$+ a_{0,1} y^{(3)}_{0,0} \cos 2\tau + y^{(3)}_{0,1} \cos 2\tau + a_{0,1} \breve{G}$

$y''_{1,1} + y_{1,1} = - a_{1,1}s_{0,0} - a_{1,0}y_{0,1}$

$- a_{0,1}y^{(3)}_{1,0} - \varepsilon_1 y^{(1)}_{1,0} \sin 2\tau + \varepsilon_2 y^{(2)}_{1,0}$

$\mathrm{Sin}2\tau + a_{1,0}y_{0,0}\cos^{(3)} 2\tau + y^{(3)}_{1,0}\cos 2\tau + a_{1,0}\breve{G}$

After substituting

$$y^{(3)}_{0,0} = a\cos\tau + b_3\sin\tau \qquad (71)$$

in the twelfth equation of the system (70) we get

$$y''^{(3)}_{0,0} + y''^{(3)}_{1,0} = - a_{1,0}(a_3\cos\tau + b_3\sin\tau) \qquad (72)$$

For the non-zero a_3 and b_3 the following results from the condition of periodicity

$$a_{1,0} = 0 \qquad (73)$$

Thence

$$y^{(3)}_{1,0} = 0 \qquad (74)$$

Making use of (71) in the first and sixth equation of the system (70), we shall obtain their particular solutions

$$y^{(1)}_{1,0} = [\frac{g}{v^2_{1,3}} + \frac{\Omega^2}{v^2_{1,3} - 1}(a_3\cos\tau + b_3\sin\tau)]$$

$$[\frac{v}{p_1} \frac{2}{2} \frac{}{1,3} - v \ \ddot{x}_{1,3}\ v_o + \omega\rho\nu\ (\nu'^2_{o})^3)] \qquad (75)$$

$$y^{(2)}_{1,0} = [\frac{g}{v^2_{2,2}} + \frac{\Omega^2_1}{v^2_{2,2} - 1}(a_3\cos\tau + b_3\sin\tau)]$$

$$[\frac{v}{p_2} \frac{2}{2} \frac{}{1,3} - v \ \ddot{x}_{2,3}\ v''_o + \omega\rho\nu\ (\nu''^2_{o})^3)] \qquad (76)$$

After substituting (71) in the thirteenth equation of the system (70), we have

$$Y''^3_{0,1} + y^{(3)}_{0,1} = \breve{G} + (- a_{1,0}\, a_3 + \frac{1}{2}a_3 + P)\cos\tau$$

$$+ (- a_{1,0}\, b_3 - \frac{1}{2}b_3 - Q)\sin\tau + \frac{1}{2}(a_3\cos 3\tau + b_3\sin 3\tau) \qquad (77)$$

The condition of periodicity gives

$$a^{(1)}_{0,1} = \frac{1}{2} + \frac{P}{a_3}$$

$$a^{(2)}_{0,1} = \frac{1}{2} + \frac{Q}{b_3} \qquad (78)$$

The particular solution of this equation is the following:

$$y_{0,1} = \breve{G} - \frac{1}{16}(a_3\cos3\tau + b_3\sin3\tau) \qquad (79)$$

When substituting (71) in the second and seventh equation of the system (70), we obtain their particular integrals:

$$y_{0,1}^{(1)} = \frac{\gamma_1}{\nu_{1,3}^2-1}\left(\frac{\nu_{1,3}^2}{2}b_3 + Q\right)\cos\tau$$
$$= \frac{\gamma_1}{\nu_{1,3}^2-1}\left(\frac{\nu_{1,3}^2}{2}a_3 + P\right)\sin\tau + \frac{\nu_{1,3}^2\gamma_1}{2(\nu_{1,3}^2-9)}$$
$$(a_3\sin3\tau - b_3\cos3\tau) \qquad (80)$$

$$y_{0,1}^{(2)} = \frac{\gamma_2}{\nu_{2,3}^2-1}\left(\frac{\nu_{2,3}^2}{2}b_3 + Q\right)\cos\tau$$
$$= \frac{\gamma_2}{\nu_{2,3}^2-1}\left(\frac{\nu_{2,3}^2}{2}a_3 + P\right)\sin\tau + \frac{\nu_{2,3}^2\gamma_2}{2(\nu_{2,3}^2-9)}$$
$$(a_3\sin3\tau - b_3\cos3\tau) \qquad (81)$$

After substitution (71) and (73) in the fourteenth equation of the system (70), we obtain the following from the condition of the existence of periodic solutions

$$a_{2,0} = 0 \qquad (82)$$

Analogously, taking (71), (79), (80), (81) into account in the fifteenth equation of the system (70), we shall obtain equation which, after transformations, will assume the form

$$a_{0,2}^{(1)} = \frac{\gamma_1\varepsilon_1\nu_1}{4}\left(\frac{\nu-1}{2} + \frac{1}{\nu_{1,3}^2-9}\right)$$
$$+ \frac{\gamma_1\varepsilon_1\nu_{1,3}^2}{4}\cdot\left(\frac{1}{\nu_{2,3}^2-1} + \frac{\nu_{1,3}^2}{\nu_{2,3}^2-9}\right)$$
$$\frac{1}{2}a_{0,2}^{(1)} - \frac{1}{32} - \frac{P}{a}\left(\frac{\varepsilon_1\gamma_1}{2(\nu_{1,3}^2-1)} + \frac{\varepsilon_2\gamma_2}{2(\nu_{2,3}^2-1)}\right) \qquad (83)$$

$$a_{0,2} = \frac{\gamma_1\varepsilon_1\nu_{1,3}^2}{4}\left(\frac{1}{\nu_{1,3}^2-1} + \frac{1}{\nu_{2,3}^2-9}\right)$$
$$+ \frac{\gamma_2\varepsilon_2\nu_{2,3}^2}{4}\cdot\left(\frac{1}{\nu_{2,3}^2-1} + \frac{1}{\nu_{2,3}^2-1}\right)$$
$$-\frac{1}{2}a_{0,2}^{(2)} - \frac{1}{32} - \frac{Q}{b}\left(\frac{\varepsilon_1\gamma_1}{2(\nu_{1,3}^2-1)} + \frac{\varepsilon_2\gamma_2}{2(\nu_{2,3}^2-1)}\right) \qquad (84)$$

The following algebraic equation system will be obtained from the condition of periodicity of the solutions of the equation system (70) after substituting (71), (74), (75), (76) and (79) in its sixteenth equation:

$$-a_{1,1}a_3 - \frac{\varepsilon_1 c_1\Omega_1}{2(\nu_{1,3}^2-1)} - \frac{\varepsilon_2 c_2\Omega_2}{2(\nu_{2,3}^2-1)}b_3 = 0$$
$$-a_{1,1}b_3 - \frac{\varepsilon_1 c_1\Omega_1}{2(\nu_{1,3}^2-1)} - \frac{\varepsilon_2 c_2\Omega_2}{2(\nu_{2,3}^2-1)}a_3 = 0 \qquad (85)$$

Where:

$$C_1: \frac{\nu}{P^2_{1,3}} - \nu^2_{1,3} \ddot{x}_1 v'_o + \omega\rho\nu^2_{1,3} (v'_o)^3$$

$$C_1: \frac{\nu}{P^2_{2,3}} - \nu^2_{2,3} \ddot{x}_1 v''_o + \omega\rho\nu^2_{2,3} (v'_o)^3$$

From the condition of a non-zero solution of the equation system (85) in relation to a_3 and b_3, we obtain

$$a_{1,1} = \pm \left[\frac{\Omega^2}{2} \left(\frac{\varepsilon c}{\nu^2_{2,3} - 1} - 2\frac{\varepsilon c}{\nu^2_{1,3} - 1} \right) \right]^{\frac{1}{2}} \qquad (86)$$

The coefficients of the sought series (67) are determined by the expressions (73), (78), (83), (84) and (86).

RESULTS

Calculation Examples

The analytically obtained results of parametric instability zones are to illustrate the influence of particular parameters of the system on their magnitude and position. The physical parameters of the system are given in the form in which they occur in the differential Eg. (12).

The influence of unbalance *μP,* damping *(μH₁),* and the shape of friction characteristics *(α/β)* on the magnitude of the parametric instability zones for p_1 and p_2, for the following data: $\Omega^2 = 900s^{-2}$, $\Omega_1^2 = 480s^{-2}$, $\omega_1^2 = 4800s^{-2}$, $g = \mu G = 9{,}81ms^{-2}$, $v_n = 0{,}4ms^{-1}$, $\varepsilon = 0{,}2$. On the basis of (14), $p_1 = 73, 32s^{-1}$ $p_2 = 28, 35s^{-1}$, and $p_3 = 69{,}28s^{-2}$ have been obtained. The adequate coefficients assume the form $\gamma_1 = 0{,}833$, $\gamma_2 = 0, 12$, $\beta_1 = 0{,}126$, $\beta_2 = -0{,}674$, $\varepsilon_1 = 1{,}176$, $\varepsilon_2 = 0{,}376$. (While, $\alpha/\beta = \alpha/\rho$).

The parameter instability zones (for p_1) and 5 (for p_2) expand with the increase of unbalance *μP,* while, depending on the value of the quotient α/β, this tendency can have different intensity. In the case of $\alpha/\beta = 0{,}5m^2s^2$ the doubling of unbalance has caused the expansion of the instability to double for zones for p_1 as well as for p_2. For $\alpha/\beta = 1m^2s^{-2}$ the increase a tripling of unbalance brings about a comparatively small expansion of the instability zones for p_1, while for p_2 the expansion is still almost doubled. In the case of large unbalance of the rotor, the changes of the quotient α/β do not influence the magnitude of the parametric instability zones. The influence of damping on the magnitude of the instability zones corresponding to the frequencies p_1 and p_2 is also very different. Small damping ($\mu H_1 = 0{,}05s^{-1}$) causes considerable shift of the zone for p_2 in the direction of the growing value of modulation depth μ($\mu \geq 0{,}15$). In the case of double increase of damping the zone will not occur for $\mu \leq 0, 3$.

The magnitude and position of the instability zones for p_1 are not sensitive to changes of the damping coefficient. In the case of $\mu H_1 = 10s^{-1}$ the pre open zone of frequency p_1 exists for ≥ 0,034. After a doubled increase of damping, when $\mu H_1 = 20s^{-1}$, the lower border of the occurrence of the zone is shifted to the value of $\mu = 0, 07$.

The parametric instability zone for p3 depends on the initial conditions of the system's motion. It has been prepared on the assumption that $a_3 = b_3 = 0{,}01m$ where $a_3 = y_3(0)$, $b_3 = y_3(0)$. The calculations, in the case of the resonance coordinate y_3 have been performed with exactitude up to the second approximation, thence the inclination of the instability zones in the direction of the growing values of the parameter λ^2_3. For the first approximation, the zones remains symmetrical in relation to the straight line $\lambda^2_3 = 1$. As in the cases considered above the increase of unbalance considerably expands the instability zone. The changes of the value of the quotient α/β and damping have a negligible influence on the magnitude of the zone. Figure 7. Presents the parametric instability zones for various values of the parameters Ω^2, Ω_1^2 and ω_1^2. For the zones denoted by 1 we get $\Omega^2 = 14400s^{-2}$, $\Omega_1^2 = 1920s^{-2}$, $\omega_1^2 = 19200s^{-2}$; for the zones denoted by 2 we have $\Omega^2 = 3600s^{-2}$, $\Omega_1^2 = 480s^{-2}$, $\omega_1^2 = 4800s^{-2}$ and for the zones denoted by 3: $\Omega^2 = 900s^{-2}$, $\Omega_1^2 = 120s^{-2}$, $\omega_1^2 = 1200s^{-2}$. In all the cases the magnitudes of the other parameters are as follows: $\mu H_1 = 10s^{-1}$, $\varepsilon = 0.2$, $\alpha/\beta = 0.5m^2s^{-2}$, $v_0 = 0.4ms^{-1}$, $\mu P = 0{,}0015m$.

The growth of the squares of frequencies Ω^2, Ω_1^2, ω_1^2 (resulting from the increase of rigidity of the elastic elements in the system or from the decrease of the values of masses) causes that the instability zones for p_1 and p_2 expand. For example, the increase of the parameters Ω^2, Ω_1^2 and ω_1^2 by four times brings about an approximately doubles expansion of the zones. The instability zone for p_3 is not influenced by the frequency changes in the system [Fig. 9(c)].

There is influence of the velocity changes of the belt v_o on the magnitude of the instability zones for p_1 and p_2. Calculations have been performed for the data denoted by 1, except for the velocity v_o, whose value has been changed.

In each case the velocity increase of the belt causes the expansion of the instability zones. For v_o < 3,3ms^{-1} these changes are less evident. The influence of the velocity changes v_o on the parametric instability zone p_3 is practically negligible.

CONCLUSION

The paper presents the analysis of a discrete mechanical system with three degrees of freedom, where self excited vibrations caused by friction, as well as parametric and forced vibrations occur. The system of ordinary differential equations governing the motion of the analyzed system if nonlinear section of the six order. The periodicity of the coefficients in the linear section of the equation of motion results from non-identical moments of inertia of the shaft cross section of the rotor consisting part of the analyzed system. The non-linearity is introduced into the equations of motion by friction between the belt and the rigid mass element where the rotor is placed. Moreover, it is increased by the normal reaction changes between the belt and the rotor base resulting from the rotor vibrations. External excitation in the form of a periodic function of time is also introduced into the system; the excitation is the effect of the unbalance of the rotor.

The analysis performed makes it possible to present the following conclusions:

(1) The method of seeking a solution as the power series of two perturbation parameters μ and ε used in the considerations makes it possible to investigate the single resonances of any order for the systems with weak nonlinearity and weakly modulated ($\mu \ll 1$).

When performing calculations with exactitude up to the second approximation, it turns out that the limits of unstability zones incline in the direction of the growing values of the parameter $\lambda^2{}_3$ (Fig. 6). For the first approximation, the limits remain symmetrical in relation to the straight line $\lambda^2{}_3 = 1$.

(2) The parametric instability zones for p_1 and p_2 expand with the increase of the rotor unbalance. Depending on the value of the quotient α/p, this tendency has different intensity. In the case of α/β = 0,5 m^{2}s^{-2}, a double increase of inbalance has brought about a considerable expansion of the instability zones, for p_1 as well as for p_2. For α/β = 0,1 m^{2}s^{-2} the unbalance which causes a rather small expansion of the instability zone for p_1 while for p_2 the expansion is still almost double. In the case of lack of the rotor unbalance the changes of the quotient α/β do not influence the magnitude of the parametric instability zones. The influence of damping on the magnitude of the instability zones corresponding to p_1 and p_2 is also very different. Minimum damping (μH_1 = 0,5 m^{2}s^{-2}) causes considerable shift of the zone for p_2 in the direction of the growing values of modulation depth (μH_1 = 0,5 m^{2}s^{-2}). The magnitude and position of the instability zones for p_1 are not so sensitive to the damping coefficient changes. The regularities indicated here are the more clear, the greater is the difference between the values of frequency p_1 and p_2 (i.e. for $p_1 \gg p_2$). The increase of unbalance also produces a considerable expansion of the instability zone for p_3; however the changes of the parameter and of damping have no essential influence on the magnitude of the zone.

The growth of the frequency squares Ω^2, $\Omega_1{}^2$ and $\omega_1{}^2$ causes the expansion of the instability zones for p_1 and p_2 are expanded. This property is noticeable within the range of great velocity (v_o > 0,3ms^{-1}). The influence of the velocity changes v_o on the parametric instability zone for p_3 is practically negligible.

(3) For the frequencies p_1 and p_2, the position of instability zone limits does not depend in the first approximation on the initial conditions of the system's motion. The magnitude of the instability zone limits for p_3 depends on them.

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