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Research Article

Perturbations of solar radiation pressure on the motion of a spheroidal satellite

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Abstract

This work studied the solar radiation pressure effects on the motion of a spheriodal artificial satellite. The incident and reflected radiation forces were taken into account. A semi-analytical technique was presented. Numerical solution was carried out to obtain the perturbation effects due to direct solar radiation pressure on LAGOES 1. The results were compared with the observations and previous works.

Keywords: Orbital motion, perturbation technique, celestial mechanics methods, solar radiation pressure, and thermal emission effects.

INTRODUCTION

Every satellite is perturbed by its interactions with the electromagnetic radiation, not only the direct radiation pressure caused by sunlight, but also the indirect radiations. Some of these indirect effects are produced by Earth's reflected radiation, thermal emission by satellite itself, and eclipses affecting portions of the orbit (Milani et al., 1987).

Many studies have been done to obtain the effects of solar radiation pressure on the artificial satellite motion taken into account the shape of the satellite (Milani et al., 1987; Fea, 1970; Fea and Smith, 1970; Vanderburgh and Kissell, 1971; Smith and Kissell, 1972). As an example, Moore, 1979, studied the orbital acceleration and the perturbation of the balloon satellite by summing over the sunlit part of each orbit.[6]

In this work, perturbation effects due to direct solar radiation pressure and the thermal emission forces affecting on a spheriodal satellite will be treated.

MATERIALS AND METHODS

The Direct Solar Radiation

The direct effect of the solar radiation on the satellite means the net acceleration resulting from the interaction (*i.e.* absorption, reflecting, or diffusion) of the sun light with each elementary surface of the spacecraft. Each photon carries an amount of momentum given by

$$\mathrm{M}_{\mathrm{om}} = \frac{E_g}{c} \qquad (1)$$

Where, M_{om} is the photon's momentum E_g is the energy of the photon (proportional to the photon frequency) and c is the velocity of light. The momentum can be exchanged during the interaction with a solid surface. So, the light behaves as a medium of material particles continuously emitted by the sun. A satellite whose surface has a reflection coefficient α, placed at a distance d from the sun and receiving the solar radiation at an angle of incidence ψ will experience an acceleration under the influence of solar radiation pressure, determined by

$$\bar{\mathrm{F}} = -\frac{\beta_1}{d^2}\bar{R}_\Theta \tag{2}$$

$$\beta_1 = \frac{A}{M}\frac{\Phi_0}{c}(1+\alpha)a_\Theta^2\cos^2\Psi \tag{3}$$

Where Φ_0 is the solar constant, c is the speed of light, a_Θ is the mean distance Earth-Sun, and $\bar{R}_\Theta$ is a unit vector in the direction Earth-Sun given in a geocentric equatorial frame by

$$\bar{R}_\Theta = \cos\Lambda_\Theta\bar{i} + \cos\varepsilon\sin\Lambda_\Theta\bar{j} + cin\,\varepsilon\sin\Lambda_\Theta\bar{k} \tag{4}$$

Where, Λ_Θ is the true celestial longitude of the Sun, ε is the obliquity of the ecliptic, Λ_Θ is expressed in terms of the orbital elements as $\Lambda_\Theta = f_\Theta + \omega_\Theta$

$$\bar{F}_i = -\beta_1\frac{\bar{R}_\Theta}{r_\Theta^2} \tag{5}$$

Where r_Θ is the mean distance satellite-Sun.

The thermal emission

Thermal emission from the satellite itself is the indirect effect of the interaction between the solar radiation and the artificial satellite. It is due to the fact that the equilibrium temperature distribution on the satellite surface is non-uniform, because of the different orientations with respect to solar heating of different parts of the spacecraft body, which causes a net force acts on the spin of the semi major axis of the artificial satellite orbit. (Sehnal et al., 1997) studied many parameters affecting the orbit of the accelerometer MACEK, they found that the acceleration due to the anisotropic thermal emission is 2×10^{-9} cm s^{-2}. Also, these effects on Mimosa satellite were found as 2.3×10^{-9} cm s^{-2} (Sehnal et al., 1998). Since the thermal photons emitted from the hotter areas of the surface carry away more momentum than those emitted from colder areas, there will be two main asymmetries of temperature distribution for a spinning satellite (Milani et al., 1987).

1-A seasonal asymmetry which arises from the fact that the angle ξ between the spin axis and the sun's direction is not 90^0 and changes with annual periodicity. This seasonal asymmetry causes a different radiation flux on the northern and southern sides of the satellite (assuming satellite as a small planet).

2-A diurnal asymmetry between the day and the night sides is presented, whenever ξ is not zero.

Now, the force due to the seasonal temperature asymmetry should be estimated. The energy flux absorbed by the satellite's surface element is given by

$$E_{absorb} = \delta\,\Phi_0\,\bar{n}.\bar{s} \tag{6}$$

Where δ is the absorption coefficient, Φ_O is the solar constant = 1.367×10^6 erg cm^{-2} s^{-1}, $\bar{n}$ is a unit vector to the surface element, $\bar{s}$ is a unit vector pointing towards the sun.

The energy emitted per unit area is given by

$$E_{em} = \varepsilon\,\sigma T^4 \tag{7}$$

Where σ = 5.67×10^{-5} erg cm^{-2} s^{-1} k^{-1} is the Stephen Boltzmann constant, T is the surface temperature, ε is the emissivity coefficient, which equals one for the black body.

The thermal state of a spherical, homogeneous, and uniformly rotating body is considered which is illuminated by the Sun. The satellite spins around the z-axis with unit vector $\overline{w}$, while the y-axis contains the projection of $\overline{s}$ on the xy plane, and $\overline{n}$ is the unit vector which is perpendicular to $\overline{s}$.

Applying the polar coordinates (r,θ,Φ) we obtain

$$\overline{n}=\left(\sin\theta\cos\Phi,\sin\theta\sin\Phi,\cos\theta\right) \tag{8.1}$$

$$\overline{s}=\left(0,\sin\zeta,\cos\zeta\right) \tag{8.2}$$

$$\overline{n}.\overline{s}=\sin\theta\cos\Phi\sin\zeta+\cos\zeta\cos\theta \tag{8.3}$$

For the steady state case the heat conduction equation takes the Laplace's form,[9] which is given by

$$\nabla^2 T=0 \tag{9}$$

Equation (9) can be solved in the form

$$T\left(r,\theta\right)=T_0+\sum_{n=1}^{\infty}T^n\left(\frac{r}{R}\right)^n P_n\left(\cos\theta\right) \tag{10}$$

Where T_0 is the average temperature, R is the radius of the sphere, T^n are constant coefficients, and $P_n\left(\cos\theta\right)$ is Legendre polynomial of order n.

As a boundary condition, the balance between the outward heat flow caused by thermal conduction which is given by $-\chi\frac{\partial T}{\partial r}$ and the emission at the surface (i.e. the difference between $\varepsilon\,\sigma T^4$ and the energy flux absorbed from the Sun $\delta\,\overline{n}.\overline{s}\,\Phi_0$), is given by

$$-\chi\frac{\partial T}{\partial r}=\varepsilon\,\sigma T^4-\delta\,\overline{n}.\overline{s}\,\Phi_0 \tag{11}$$

Where χ is the conductivity. It is more convenient to express $\overline{n}.\overline{s}$ in terms of its angular average which is given by

$$S(\theta)\frac{1}{2\pi}\int \overline{n}.\overline{s}\,d\Phi \tag{12}$$

The integral is extended only to the illuminated hemisphere, i.e. for $\overline{n}.\overline{s}\succ 0$. If ζ is chosen to be less than $\frac{\pi}{2}$, the analytical expression of $S(\theta)$ is given by

$$S(\theta)=\cos\zeta\cos\theta \qquad for\ \theta\le\frac{\pi}{2}-\zeta \tag{13.1}$$

$$S(\theta)=\frac{\sin\zeta\sin\theta\cos\Phi_1}{\pi}+\frac{\cos\zeta-\cos\theta}{2\pi} \qquad for\quad \frac{\pi}{2}-\zeta\le\theta\le\frac{\pi}{2}+\zeta \tag{13.2}$$

$$S(\theta)=0 \qquad for\ \theta\ge\frac{\pi}{2}+\zeta \tag{13.3}$$

Where Φ_1 is the solution of $\overline{n}.\overline{s}=0$, *i.e.* $\Phi_1=-\arcsin\left(\cos\theta\cos\zeta\right)$

In order to derive the force due to anisotropic thermal emission, it is considered that each surface element emits in the direction $\overline{n}$ a momentum flux as

$$F_R = -\left(\frac{2\varepsilon\sigma T^4}{3c}\right)\bar{n} \qquad (14)$$

Where c is the velocity of light, the factor (2/3) is allowed for the fact that the emission is assumed to follow Lambert's law. This emission corresponds to a net force acting on the satellite which is given by

$$F_R = -\left(\frac{2\varepsilon\sigma T^4 ds}{3c}\right)\bar{n} \qquad (15)$$

Where ds is the area of the surface element. The total force is obtained by integrating over the whole body surface. Owing to the axial symmetry of the problem, it is directed along W , the explicit computation yields

$$F_R = -W\left(\frac{4\pi}{9c\beta}\delta\Phi_0 R^2\cos\zeta\right) \qquad (16)$$

β is reduction coefficient given by

$$\beta = 1 + \frac{\chi T_0}{\delta R \Phi_0} \qquad (17)$$

On the other hand, the diurnal temperature asymmetry which causes a force perpendicular to the spin axis can be neglected, since this force is proportional to the temperature difference, it is very small in comparison with that arising from the seasonal effect.

The effects of both the incident and reflected solar radiation pressure forces on a spheroidal satellite could be obtained taken into account the following assumptions:

i) The sun moves in a circular orbit such that $\mathrm{A}_{\odot}$ becomes the mean longitude of the Sun, *i.e.* $vt + cons\tan t$;
ii) The direction and distance of the satellite from the Sun are similar to those of the Earth;
iii) The Sun-satellite line is parallel to the Sun Earth line;
iv)The satellite has a high reflecting surface, so that the diffuse component of the reflected radiation is negligible;
v) The albedo effect is ignored.

Now, if the satellite is a slowly spinning or a three axis stabilized spacecraft, equation (16) would not be valid. In these cases the satellite's thermal behavior would be very different due to the complex shape and the active thermal system. It is known that, if the temperature difference ΔT between two substantial parts or sides of the body is existed, the perturbed acceleration can be evaluated from the expression (Milani et al., 1987).

$$F_R = \frac{4}{9}\delta\frac{A}{M}\frac{\Phi_0}{c}\frac{\Delta T}{T_0} \qquad (18)$$

Where $\frac{A}{M}$ is the mean value of area to mass ratio of the satellite, Equation (18) is considered the general formula to obtain the force due to the anisotropic thermal emission.

Finally, the effects of both the incident and reflected solar radiation pressure forces on a spheroidal satellite can be obtained from Equations (5) and (18) in the form

$$F(s) = \frac{A}{M}\frac{\Phi_0}{c}\left(\frac{9}{4}\delta\frac{\Delta T}{T} - (1+\alpha)\frac{a_{\odot}^2}{d^2}\cos^2\psi\bar{R}_{\odot}\right) \qquad (19)$$

Equations of motion

For a spinning satellite, there will be two main temperature asymmetry distributions. Assuming the satellite as a small planet, then, a seasonal temperature asymmetry will arise due to the fact that the angle ξ between the spin axis and the Sun's direction is not 90^{o} and changes with annual periodicity, causing a different radiation flux on the northern and southern sides of the satellite.

Therefore, the acceleration due to the seasonal temperature asymmetry will be directed along the satellite's spin axis, and it is a function of the angle between the spin axis itself (W) and the Sun's direction (s). If w is assumed to be constant, (*i.e.* if the satellite is rotating around the axis of maximum moment of inertia and no free or forced precession is present), and if there is no eclipse, then this acceleration is a function only of the variable s. Then the acting force will be a function of s with three components $F_x(s), F_y(s), F_z(s)$.

The variations of the orbital elements due to the perturbations of the total incident radiations and thermal emission will be only contributed by the radial RR and the transverse TR components. In an orthogonal reference frame with the origin at the center of the Earth, x-axis is pointing towards the perigee of the satellite orbit and z-axis is directed as the satellite orbital angular momentum, the radial, in-plane transverse, and out of plane unit vectors are written as

$$q = (\cos f, \sin f, 0) \qquad (20.1)$$

$$tf = (-\sin f, \cos f, 0) \qquad (20.2)$$

$$w = (0,0,1) \qquad (20.3)$$

Where, f is the true anomaly

$$RR = F(s).q \qquad (21.1)$$

$$TR = F(s).tf \qquad (21.2)$$

$$W = F(s).w \qquad (21.3)$$

Substituting into the Lagrange equations Gaussian form, for simplicity $F(s)$ replaced by F, after some reductions this yields (Aksnes, 1976).

$$\delta a = 2na^3\left(1-e^2\right)^{-\frac{1}{2}} F\left[eRR\sin f + TR\,\frac{a\left(1-e^2\right)}{r}\right] \qquad (22.1)$$

$$\delta e = na^2\left(1-e^2\right)^{\frac{1}{2}} F\left\{RR\sin f + TR\left[\cos f + \frac{1}{e}\left(1-\frac{r}{a}\right)\right]\right\} \qquad (22.2)$$

$$\delta i = na^2\left(1-e^2\right)^{-\frac{1}{2}} FW\,\frac{r}{a}\cos u \qquad (22.3)$$

$$\delta\Omega\sin i = na^2\left(1-e^2\right)^{-\frac{1}{2}} FW\,\frac{r}{a}\sin u \qquad (22.4)$$

$$\delta\omega = -\delta\Omega\cos i + \frac{na^2\left(1-e^2\right)^{\frac{1}{2}}}{e} F\left[-RR\cos\upsilon + TR\left(1+\frac{r}{p}\right)\sin i\right] \qquad (22.5)$$

$$\delta M = n - na^2 F\,RR\,\frac{r}{a} - \left(1-e^2\right)^{\frac{1}{2}}\left(\delta\omega + \delta\Omega\cos i\right) \qquad (22.6)$$

Where f is the true anomaly, $u = f + \omega$, F denotes the magnitude of the radiation pressure force per unit satellite mass, p = 4.65 x 10^{-5} is the force per unit area which exerted at the Earth by the Sun when its geocentric distance equals its mean distance (*i.e.* $R_{\odot} = a_{\odot}$), and W are the direction cosines of the force F along the satellite's radius vector r .

Effect of the earth's shadow

An eclipse occurs when the satellite enters the area projected by the Earth's shadow on the satellite's orbit. This area is an ellipse, whose semi-major axis and semi-minor axis are defined as, $R \cos ec\, I_{\odot}$, and R , where $I_{\odot}$ is the geocentric angel between the Sun and the orbital plane (Figure 1).

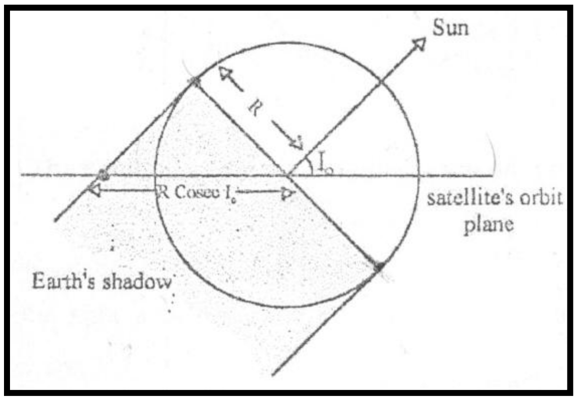

Figure1. Section normal to the orbital plane containing the Sun position

Then, from the equation of an ellipse with the center of the ellipse is the point of origin, the Earth's shadow is given as

$$r_{sh}^2 = \frac{R^2}{1 - e_{sh}^2 \cos^2 \theta} \qquad (23)$$

Where, θ is measured from the conjunction point in the orbital plane, it therefore has two values, the entrance or the exit of the satellite into or from the Earth's shadow (Figure 2).

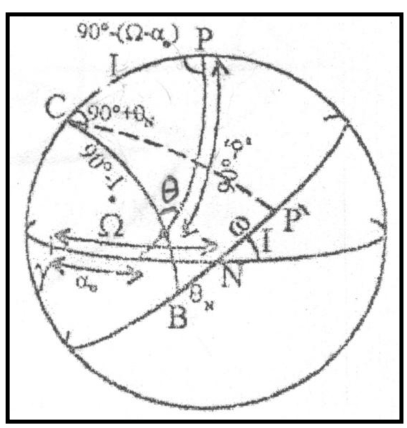

Figure 2. The position of the Sun relative to the orbital plane

From the spherical triangle CPθ

$$\sin I_{\odot} = \cos I \sin \delta_{\odot} + \sin I \cos \delta_{\odot} \sin\left(\Omega - a_{\odot}\right); \; -90^0 \le I_{\odot} \le 90^0 \qquad (24.1)$$

Where, $a_{\odot}$, and $\delta_{\odot}$ are the right ascension and declination of the Sun at the time of nearest perigee passage. Then

$$\cos\theta_N = \frac{\cos\delta_{\odot}\ \cos\left(\Omega - a_{\odot}\right)}{\sin I_{\odot}} \qquad (24.2)$$

$$\sin\theta_N = \frac{\cos I\ \sin I_{\odot} - \sin\delta_{\odot}}{\sin I\ \cos I_{\odot}} \qquad (24.3)$$

The geocentric angle θ_ω in the orbital plane between the perigee and the conjunction point (as in Figure 3), is given by:

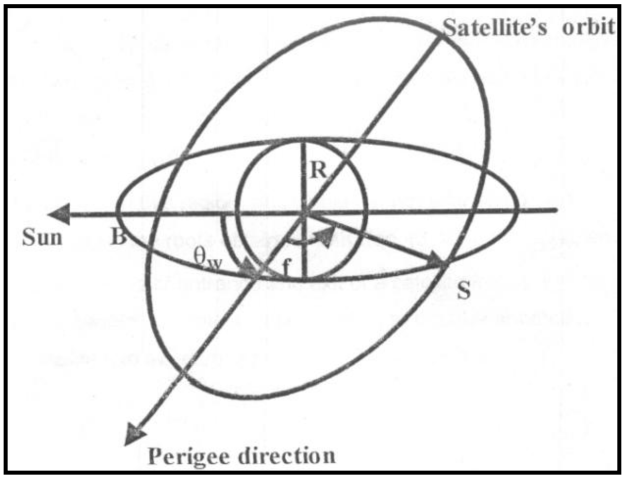

Figure 3. The angle between the perigee and the conjunction point

$$\theta_\omega = \theta_N + \omega \qquad (25.1)$$

Then, the angle between the conjunction point and the satellite is

$$\theta = \theta_\omega + f \qquad (25.2)$$

Now, the satellite enters and leaves the shadow when $r = r_{sh}$

Then, from Equation (23) and by using Equations (24) and (25) the shadow equation will be

$$k_1 \cos^2\left(\theta_\omega + f\right) + k_2 \cos^2 f + k_3 \cos f + k_4 = 0 \qquad (26)$$

Where

$$k_1 = a^2\left(1 - e^2\right)^2 \cos^2 I_{\odot} \qquad (27.1)$$

$$k_2 = R^2 e^2 \qquad (27.2)$$

$$k_3 = 2e R^2 \qquad (27.3)$$

$$k_4 = R^2 a^2 \left(1 - e^2\right)^2 \qquad (27.4)$$

Equation (26) has four roots, satisfying

$$90^0 \le \theta_\omega + f \le 90^0 \qquad (28)$$

If this region has no roots, an eclipse will not take place.

Since the true anomalies of entrance and exit obtained, then the corresponding eccentric anomalies E_1 and E_2 can be obtained.

The application of this work can be obtained as follow:

(1) The eccentric anomaly E_1 for entrance into the shadow and the eccentric anomaly E_2 for the exit from the shadow are calculated from Equations (26) and (28).
(2) The variations of the elements for a satellite moving in sunlight are obtained by integrating Equations (22) from E_1 to E_2[11] this yields;

$$\delta a = 2a^3 F \left| RR\cos E + TR\left(1-e^2\right)^{\frac{1}{2}}\sin E \right|_{E_1}^{E_2} \tag{29.1}$$

$$\delta e = a^2\left(1-e^2\right)^{\frac{1}{2}} F \left| \frac{1}{4}RR\left(1-e^2\right)^{\frac{1}{2}}\cos 2E + TR\left(\frac{3}{2}E - 2e\sin E + \frac{1}{4}\sin 2E\right) \right|_{E_1}^{E_2} \tag{29.2}$$

$$\delta i = a^2\left(1-e^2\right)^{-\frac{1}{2}} FW \left| \begin{matrix} \left(-\frac{3}{2}eE + \left(1+e^2\right)\sin E - \frac{e}{4}\sin 2E\right)\cos\omega + \left(1-e^2\right)^{\frac{1}{2}} \\ \left(\cos E - \frac{e}{4}\cos 2E\right)\sin\omega \end{matrix} \right|_{E_1}^{E_2} \tag{29.3}$$

$$\delta\Omega\sin i = a^2\left(1-e^2\right)^{-\frac{1}{2}} FW \left| \begin{matrix} \left(-\frac{3}{2}eE + \left(1+e^2\right)\sin E - \frac{e}{4}\sin 2E\right)\sin\omega - \left(1-e^2\right)^{\frac{1}{2}} \\ \left(\cos E - \frac{e}{4}\cos 2E\right) \end{matrix} \right|_{E_1}^{E_2} \tag{29.4}$$

$$\delta\omega = -\delta\Omega\cos i + \frac{a^2F\left(1-e^2\right)^{\frac{1}{2}}}{e} \left| \begin{matrix} RR\left(-\frac{3}{2}E - 2e\sin E + \frac{1}{4}\sin 2E\right) + \\ TR\left(1-e^2\right)^{-\frac{1}{2}}\left(e\cos E - \frac{1}{4}\cos 2E\right)\sin\omega \end{matrix} \right|_{E_1}^{E_2} \tag{29.5}$$

$$\delta M = -\left(1-e^2\right)^{\frac{1}{2}}\left(\delta\omega + \delta\Omega\cos i\right) - 3a^2F \left| \begin{matrix} RR\left(-\frac{3}{2}eE + \left(\frac{5}{3} + \frac{2}{3}e^2\right)\sin E - \frac{5}{12}e\sin 2E\right) - \\ TR\left(1-e^2\right)^{\frac{1}{2}}\left(e\cos E - \frac{1}{4}\cos 2E\right) - \\ \left(E - e\sin E\right)\left(S\cos E_1 + TR\left(1-e^2\right)^{\frac{1}{2}}\sin E_1\right) \end{matrix} \right|_{E_1}^{E_2} \tag{29.6}$$

Where $RR = RR(0),\ TR = TR(0)$, and W could be obtained with $f = 0$, from

$$\begin{Bmatrix} RR \\ TR \end{Bmatrix} = -\cos^2\frac{i}{2}\cos^2\frac{\varepsilon}{2}\begin{Bmatrix}\cos \\ \sin\end{Bmatrix}(\lambda_{\odot} - u - \Omega) -$$

$$\sin^2\frac{i}{2}\sin^2\frac{\varepsilon}{2}\begin{Bmatrix}\cos \\ \sin\end{Bmatrix}(\lambda_{\odot} - u + \Omega) -$$

$$\frac{1}{2}\sin i\,\sin\varepsilon\left[\begin{Bmatrix}\cos \\ \sin\end{Bmatrix}(\lambda_{\odot} - u) - \begin{Bmatrix}\cos \\ \sin\end{Bmatrix}(-\lambda_{\odot} - u)\right] -$$

$$\sin^2\frac{i}{2}\cos^2\frac{\varepsilon}{2}\begin{Bmatrix}\cos \\ \sin\end{Bmatrix}(-\lambda_{\odot} - u + \Omega) -$$

$$\cos^2\frac{i}{2}\sin^2\frac{\varepsilon}{2}\begin{Bmatrix}\cos \\ \sin\end{Bmatrix}(-\lambda_{\odot} - u - \Omega)$$

(30.1)

$$W = \sin i\,\cos^2\frac{\varepsilon}{2}\sin(\lambda_{\odot} - \Omega) - \sin i\,\sin^2\frac{\varepsilon}{2}\sin(\lambda_{\odot} + \Omega) - \cos i\,\sin\varepsilon\,\sin\lambda_{\odot} \quad (30.2)$$

Where ε is the obliquity, and $\lambda_{\odot}$ is the ecliptic longitude of the Sun (Kittel and Kroemer, 1980).

RESULTS AND DISCUSSION

To show the effects of the total incident radiations and thermal emission, applications on the artificial satellite ***LAGEOS*** were done. This satellite is a sphere with a radius of 30 cm, a mass of 400 kg, and $a = \varepsilon = 0.4$, typical values for unpolished metal surface. In this case the conductivity of different surfaces is shown in Table 1. (Milani et al.,1987).

Table 1. Conductivity and the reduction coefficient for different surfaces

χ **erg cm^{-1} s^{-1} k^{-1}**	β
2.1×10^7	471 for homogeneous Aluminum.
3.5×10^6	79 for homogeneous lead.
0.0	155 for core of 25 cm and outer shell of Aluminum.

It is found from equation (18) that, the semi major axis variation of ***LAGEOS*** is 2×10^{-10} cm s^{-1} due to these effects, with ΔT = 0.2 k, area to mass ratio $\frac{A}{M}$ = 0.007 cm^2 g^{-1}, and absorption coefficient a = 0.2. These effects are too small to be account, but can not be completely neglected, taking into account that the initial surface temperature T_0 is 280 k.

The diurnal temperature asymmetry causes a perpendicular force to the spin axis. In our applications the spin rate of **LAGOES** is about 10 rad s^{-1},[12] the maximum temperature difference ΔT between the day and night sides of the satellite cannot exceed 10^{-2} k [13] with a phase close to 45^0. Since the force is proportional to the temperature difference, then this force is very small in comparison with that arising from the seasonal effect. Therefore the effect of the diurnal temperature asymmetry can be neglected.

Then, the effects of the direct incident solar radiation pressure and shadow were introduced, in addition to the thermal effects; the code is constructed to solve equations (29), which give the variations in the semi major axis and also the variations in the other orbital elements. These variations were found to be 10^{-6} cm per revolution in the semi major axis.

Through up this work, we illustrated the effects of the direct solar radiation pressure and anisotropic thermal emission taken into account the Earth's shadow effects. The mean variations per one revolution were obtained for the first step of calculation, then the whole variations were presented in Figures 4-9, which illustrate these variations in the orbital elements $a, e, i, \Omega, \omega,$ and M respectively.

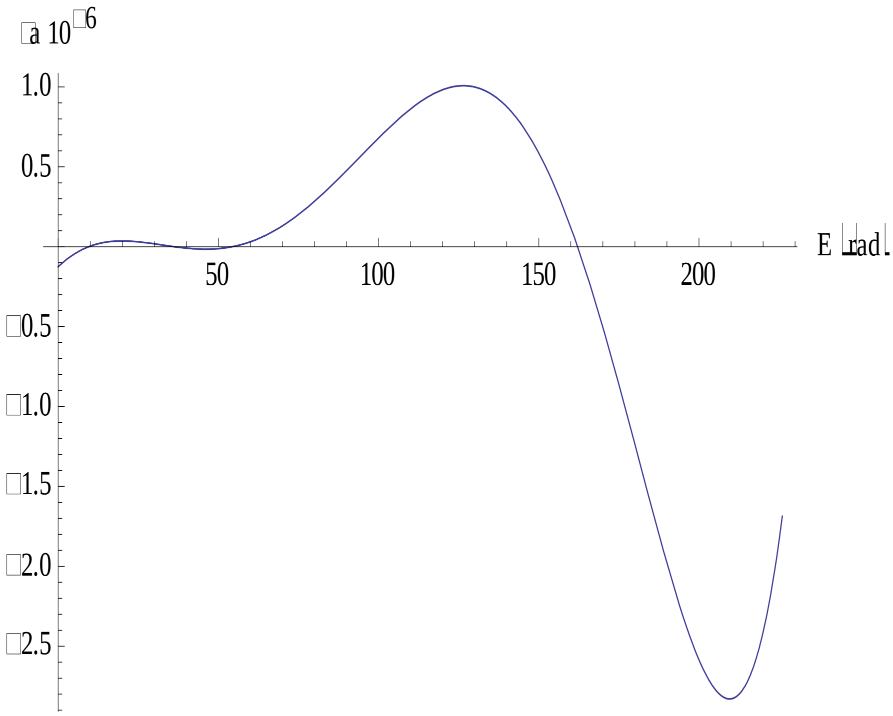

Figure 4. The variations in the semi-major axis (δa) with eccentric anomaly (E)

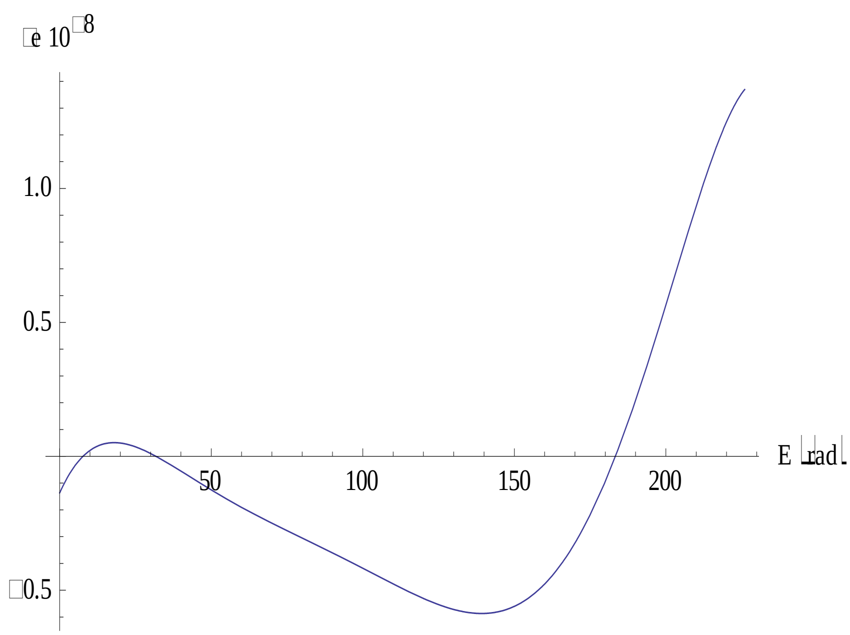

Figure 5. The variations in the eccentricity (δe) with eccentric anomaly (E)

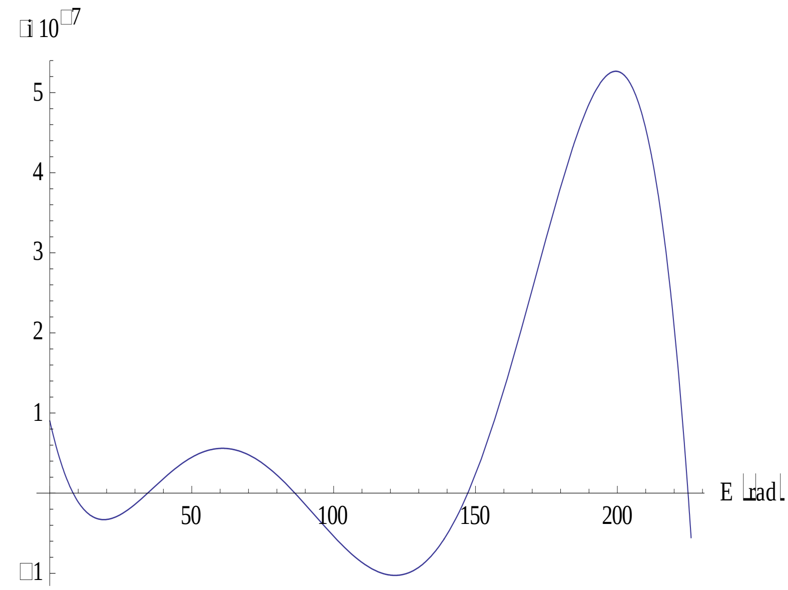

Figure 6. The variations in the inclination (δi) with eccentric anomaly (E)

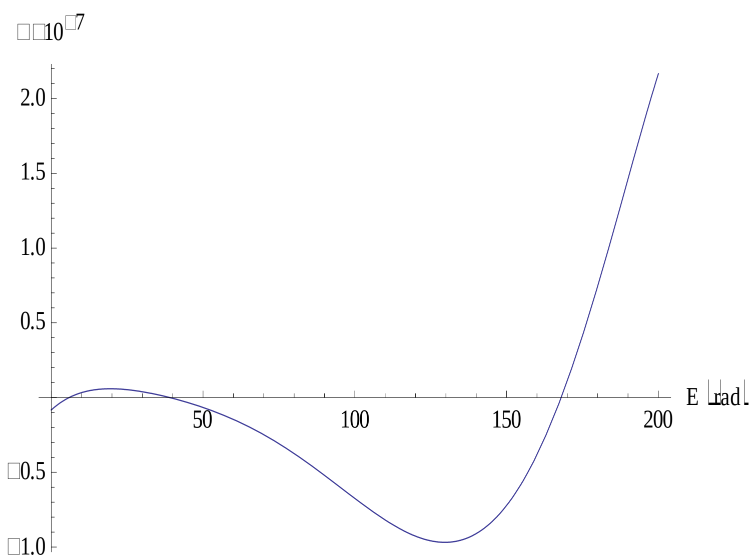

Figure 7. The variations in the argument of perigee (δ Ω) with eccentric anomaly (E)

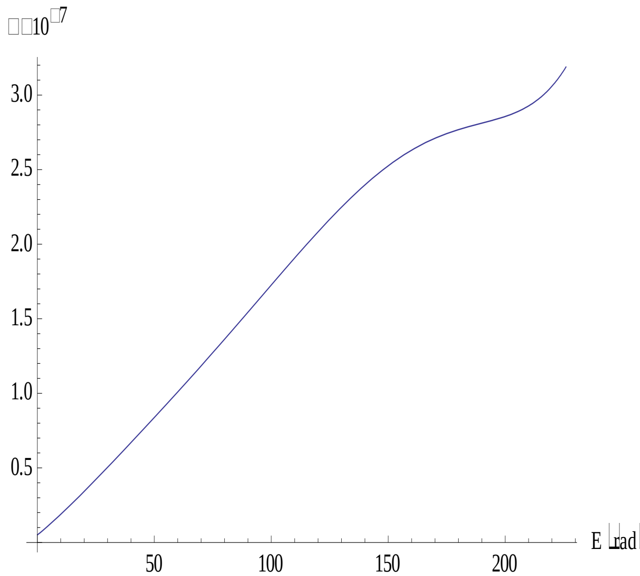

Figure 8. The variations in the longitude of the ascending node (δω) with eccentric anomaly (E)

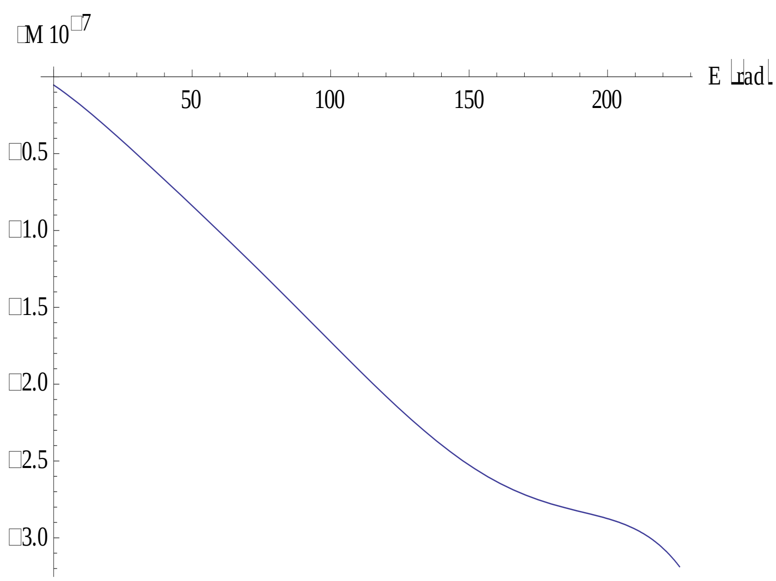

Figure 9. The variations in the mean anomaly (δM) with eccentric anomaly (E)

CONCLUSIONS

In this work the variation of orbital elements of high altitude satellite due to direct solar radiation pressure and the isotropic thermal effects were estimated. The isotropic thermal effects produce variations of order 10^{-10} which considered very small compared with that of direct solar radiation pressure 10^{-6}, so the addition of both of these effects was in agreement with the observation of the orbital elements.

In the near future the effects of the Earth's gravitational force will be studied in addition to solar radiation pressure and thermal effects, and the Earth's shadow will be taken into considerations.

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