

Research Article

Design of infinite impulse response (IIR) bandpass filter structure using particle swarm optimization

Ejike C. Anene* and Johnson Otitolaiye

Department of Electrical Engineering, Abubakar Tafawa Balewa University, PMB 0248
Bauchi, Nigeria.

*Corresponding Author Email: ejikeanene@yahoo.com

Received 16 January 2013; Accepted 21 February 2013

Abstract

This article presents the application of particle swarm optimization (PSO) algorithm for the design of infinite impulse response (IIR) bandpass digital filter. Bilinear transform (BLT) design method was first used as baseline method to compute the filters' approximate transfer functions based on conversion of the analog filter's transfer function from the s-plane to the equivalent digital filter transfer function in z-plane. The filter transfer function was obtained from the BLT design based on the given specifications. PSO algorithm was also used to generate the optimal filter coefficients for the same specifications. A population of 20 particles was initialized to probe the search space. The particles' performance was evaluated by the fitness function defined as the mean square error between the magnitude of the designed filter and the magnitude of the desired (ideal) filter. The particle's velocity and position in the search space are adjusted according to the PSO equations. The response of the PSO designed filter was compared with the response of BLT designed filter. Simulation results show close performance of both filters in discriminating a multi-frequency input signal.

Keywords: Digital filter, particle swarm optimization, fast Fourier transform, fitness function, bilinear transformation.

INTRODUCTION

Digital filters belong to the class of linear time invariant systems, which are characterized by the properties of causality, stability and recursive properties (Tan, 1999). They can be characterized in the time domain by their unit-impulse response and in the frequency domain by their transfer function. Filters are basically described by their frequency response as lowpass, highpass, bandstop or bandpass filters.

The problem addressed in this work is to investigate the use of particle swarm optimization technique to design the infinite impulse bandpass filter and compare its response with that designed using the classical Bilinear transform method. The Bilinear transformation (BLT) design technique is a well-known, well established design method used universally to design Infinite Impulse Response filters. It has yielded classical solutions for decades even though it suffers a basic deficiency of frequency warping and gain distortion. In the Bilinear transformation design technique the analog filter transfer function from the s-plane is transformed to the digital filter transfer function in the z – plane. This technique can be used to design digital filters with Butterworth or Tschebyscheff approximations for IIR filters.

Computational intelligent algorithms are capable of handling non-differentiable, non-linear and multi -objective function optimization problems. They are therefore suitable for digital filter design. PSO and its Hybrid differential evolution particle swarm optimization (DE-PSO) were applied to design an FIR filter in a work done by Bipul(2008). Kazhar and Khaled(2006) applied rational based PSO to design vectored filter whose coefficients approximate the response of a color image interpolator. The main merits of this interpolator are its edge preserving capabilities and the absence of artifacts normally associated with classical linear and even some nonlinear interpolation schemes. Multi -objective function PSO has been applied to design an optimal hybrid power filter compensator. Filter parameters such as ω_p , ω_s , A_{\max} and A_{\min} were combined as a single objective (fitness) function (Adel et al., 2009). A variant of PSO with Perturb velocity has been applied for the design of two dimensional zero-phase IIR filter. The stability criteria were incorporated as constraints to the minimization task fitness function (Swagatam et al., 2005). The filter thus obtained has a reasonably good stability margin. PSO technique because of its efficiency has also been applied to design optical finite impulse response (FIR) filters (Ying et al., 2003). Particle swarm optimization has equally been applied for nonlinear parameter identification of digital filters (Omizegba et al., 2010). Its attractiveness for this application comes from its high efficiency with few parameters to adjust for good performance.

The study presents the design of the bandpass filter structure using the bilinear technique applied to the given specifications, the PSO design for the same specifications and the resultant transfer function. It also presents the filter responses and results discussion. Finally the summary, conclusion and the references were also given.

BILINEAR TRANSFORM DESIGN TECHNIQUE

The specification for the bandpass filter is as follows: Design a bandpass filter of sampling frequency of 1 kHz with a 3dB passband frequency from 100-400Hz and at least 20dB attenuation at 45 and 450Hz with monotonic roll off beyond these frequencies. For the classical design, the digital frequencies for the lowpass filter are obtained in line with BLT design procedure and converted to bandpass using the transformation.

The design procedure includes the following steps: (1) transforming digital filter specifications into analog filter specifications, (2) performing analog filter design, and (3) applying bilinear transformation and verifying its frequency response. For the bilinear transformation, given the digital filter frequency specifications, first prewarp the digital frequency specifications to the analog frequency specifications and perform the prototype transformation using the analog lowpass prototype to analog bandpass.

$$H_p(s) = \frac{s^2 + \omega_0^2}{sW}, \quad \omega_0 = \sqrt{\omega_l \omega_h}, \quad \omega = \omega_h - \omega_l$$

$$H(z) = H(s) \Big|_{s = \frac{z-1}{z+1} \frac{2}{T}} \quad (\text{Li, 1999})$$

The digital frequencies were obtained as follows;

$$\omega_d h = 2\pi \times 400 \text{ rad/s} = 800\pi \text{ rad/s}$$

$$\omega_d l = 2\pi \times 100 \text{ rad/s} = 200\pi \text{ rad/s}$$

$$T = \frac{1}{f_s} = \frac{1}{1000}$$

f_s is the sampling frequency $\omega_d h$ and $\omega_d l$ are the upper and lower pass band digital frequencies. The prewarp analog frequencies were obtained as follows:

$$\omega_a h = \frac{2}{T} \tan \frac{\omega_d h T}{2} = 2000 \tan \left(\frac{800\pi/1000}{2} \right) = 6155.36 \text{ rad/s}$$

$$\omega_a l = \frac{2}{T} \tan \frac{\omega_d l T}{2} = 2000 \tan \left(\frac{200\pi/1000}{2} \right) = 649.8 \text{ rad/s}$$

The filter order was obtained using the Butterworth approximation formula in the equation

$$n \geq \frac{\log_{10} \left(\frac{10^{0.1A_s}}{\varepsilon^2} - 1 \right)}{2 \log_{10} (V_s)}$$

Where, n is the order of the filter and;

$$V_s = \frac{\omega_{as} h - \omega_{as} l}{\omega_{ap} h - \omega_{ap} l}$$

Substituting the given passband and stopband attenuation figures into the equation to yield,

$$n = \frac{\log_{10} \frac{10^{0.1 \times 20} - 1}{1}}{2 \log_{10} 2.5} = 1.256, \text{ the order of the filter is approximated to 2.}$$

Choosing the second order Butterworth prototype transfer function combined with the analog transform prototype function for bandpass filter we obtain;

$$H(s) = \frac{1}{\left(\frac{s^2 + \omega_0^2}{SW} \right)^2 + 1.4142 \left(\frac{s^2 + \omega_0^2}{SW} \right) + 1}$$

Substituting for s from the bilinear function $H(z) = H(s) \Big|_{s = \frac{z-1}{z+1} T}$. The transfer function computed from the design for the bandpass filter specifications is then given by;

$$H_{Bp} = \frac{0.3516 - 1.0484Z^{-2} + 0.8324Z^{-4}}{1 - 0.8938Z^{-2} + 0.3112Z^{-4}} \quad (1)$$

PSO BASED DESIGN

Table 1 lists the PSO parameters used for the design of the filter structures considered. Making the acceleration constant c_1 and c_2 equal has been shown to be a good starting point while the inertia constant can start typically at 0.9 and then decreased as the iteration increases (De Valle et al., 2008). In selecting the PSO equation parameters typical values of constants shown in Table 1 were chosen for simplicity. The fitness function adopted in this work is defined as the mean square error between the magnitude of the designed filter and the magnitude of the desired filter (Bipul, 2009).

Table 1. Summary of PSO parameters.

Parameter	Value
C_1	2
C_2	2
Max. Velocity	15
Particle number	20
Number of Iterations	200
Particle dimension	5
Inertia weight(W)	0.8

This is sufficient because the design objective is centered on the frequency response of the designed filter. The fitness function J is given by Equation (2) and h is equal to the folding frequency.

$$J = \frac{1}{h} \sum_i^h (m(k)_{designed} - m(k)_{desired})^2 \quad (2)$$

The ideal filter is made very simple having a magnitude of 1 at the passband and a magnitude of 0 at the stopband. The transfer function obtained using PSO design for the bandpass is shown in equation (3).

$$H(z) = \frac{0.4516 - 1.0484z^{-2} + 0.6324z^{-4}}{1 - 0.8938z^{-2} + 0.3112z^{-4}} \quad (3)$$

FILTER RESPONSES AND DISCUSSION

Responses obtained from BLT classical design method and PSO algorithms are presented. Simulated multi-frequency sinusoidal input signal given by Equation (4) was used to test both designed bandpass filter structures.

$$\begin{aligned} f &= 50; \\ w &= 2 * \pi * f; \end{aligned}$$

$$\begin{aligned} x &= \sqrt{2} \sin(\omega n T) + \sqrt{2} \sin(2\omega n T) + \sqrt{2} \sin(8\omega n T) \\ &+ \sqrt{2} \sin(0.1\omega n T) + \sqrt{2} \sin(9\omega n T) + \sqrt{2} \sin(5\omega n T) \end{aligned} \quad (4)$$

Figure 1 shows the plot of the mean square error of the PSO in 500 epochs. The algorithm converged in about the 140th iteration to the optimal coefficients of the filter. Magnitude frequency response of the PSO designed filter compared with the ideal filter is shown in Figures 2 to 5 for both designs. The magnitude response of the PSO designed filter compared with the ideal bandpass filter is shown in Figure 2. It is noteworthy that the designed filter is maximally flat at the passband showing characteristics of Butterworth filters. The figure also shows the -3dB point at 90-420Hz which is close to the 100-400Hz passband specified.

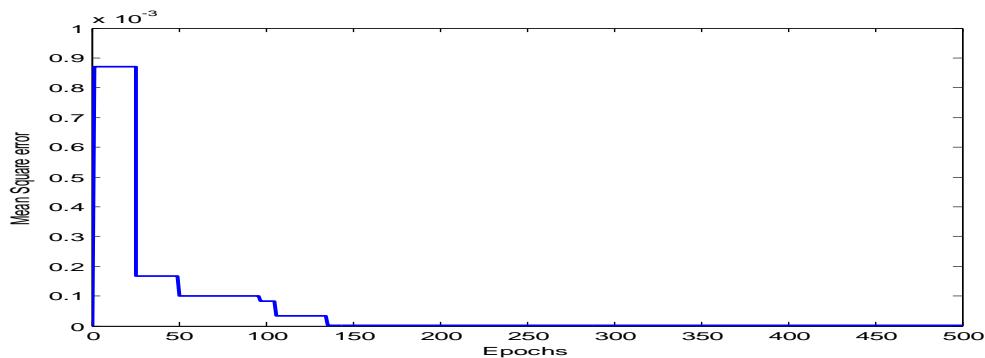


Figure 1. MSE graph for bandpass filter.

Response of the filter obtained from BLT technique is presented in Figure 4. This figure shows that the design by BLT does not closely approximate the ideal case. The response is not monotonic at the stopband and has some noticeable ripple at the passband. The -3dB point is at 50-450Hz which is far removed from the 100Hz lower passband frequency.

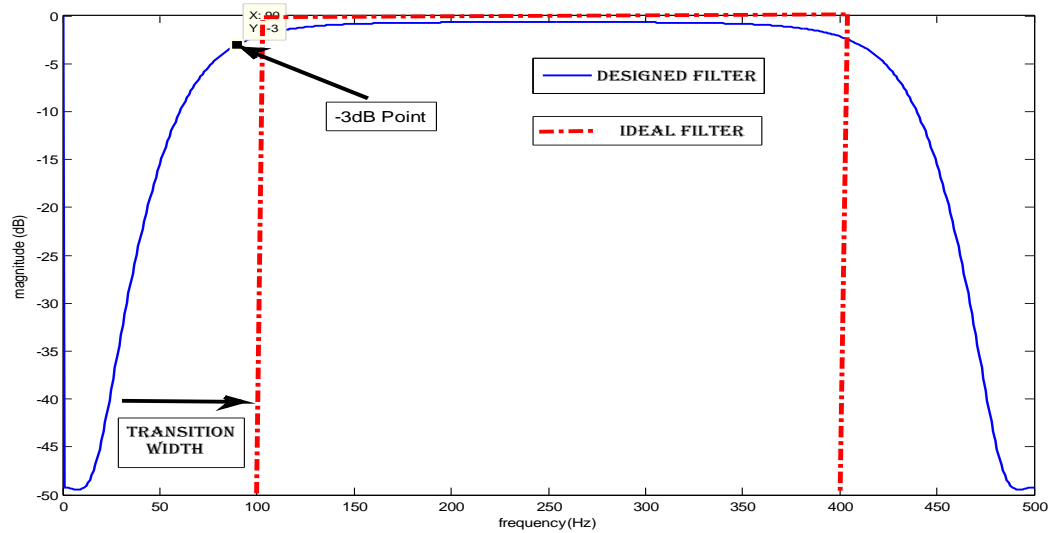


Figure 2. Gain plot of bandpass generated using PSO.

The phase responses of the filters designed by PSO and BLT are given in Figures 3 and 5. Both show the characteristics of non-linearity that is known for IIR filters.

The FFT plot of the multi-frequency test signal in Figure 6 shows the various bands of frequencies used to test the ability of both filters to discriminate unwanted frequency bands.

Inspecting Figures 7 and 8 closely, signals from the lower passband frequency of 100Hz to the upper passband frequency of 400Hz are seen to pass through the output of both filters with very little attenuation while signals at the cutoff frequencies of 50 and 450Hz are greatly attenuated. Yet for the BLT designed filter, the signal at the 50 and 450Hz frequency band, show a marked increase in gain when compared with that designed by PSO. It can be observed thus that the attenuation of the BLT designed filter is poorer.

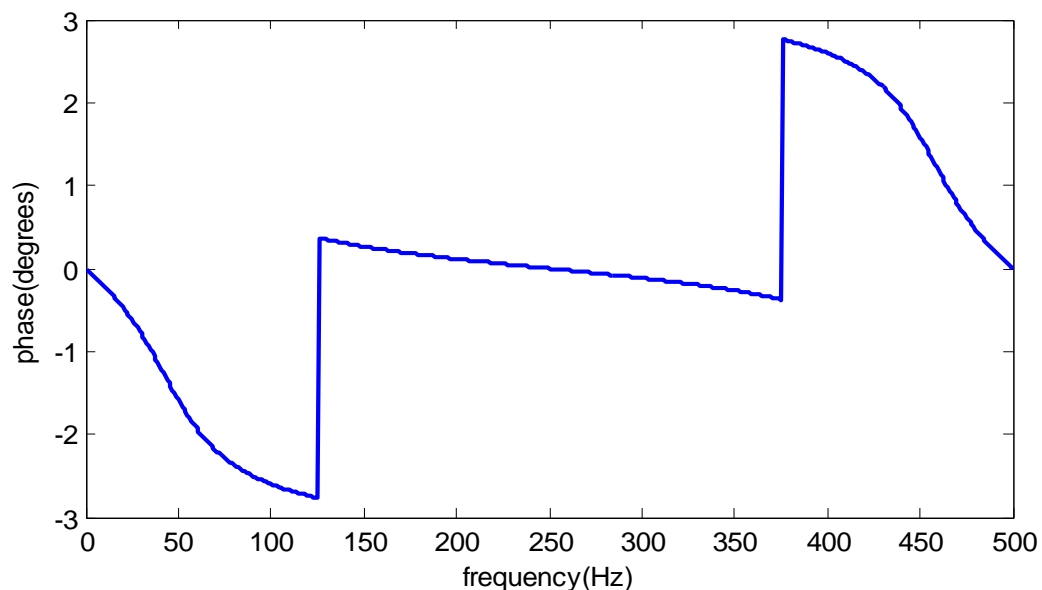


Figure3. Phase plot of bandpass generated using PSO.

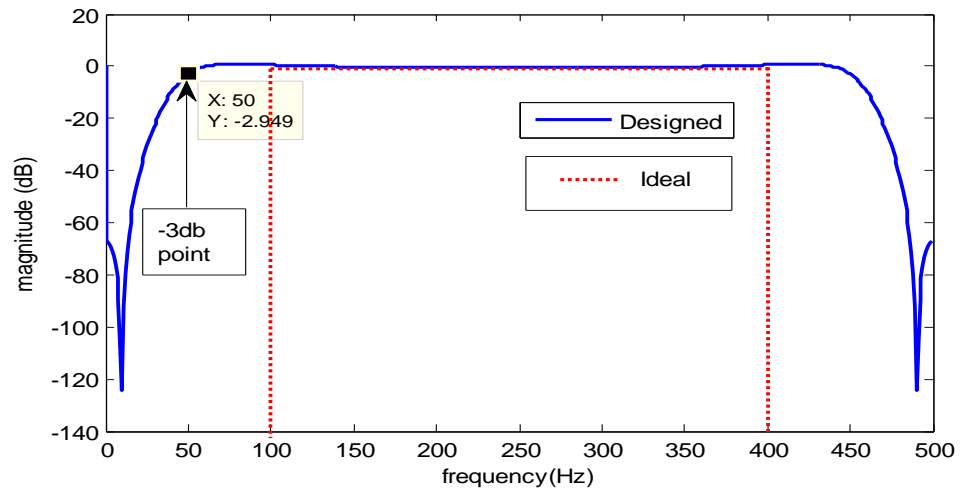


Figure 4. Gain plot of Bandpass generated using BLT.

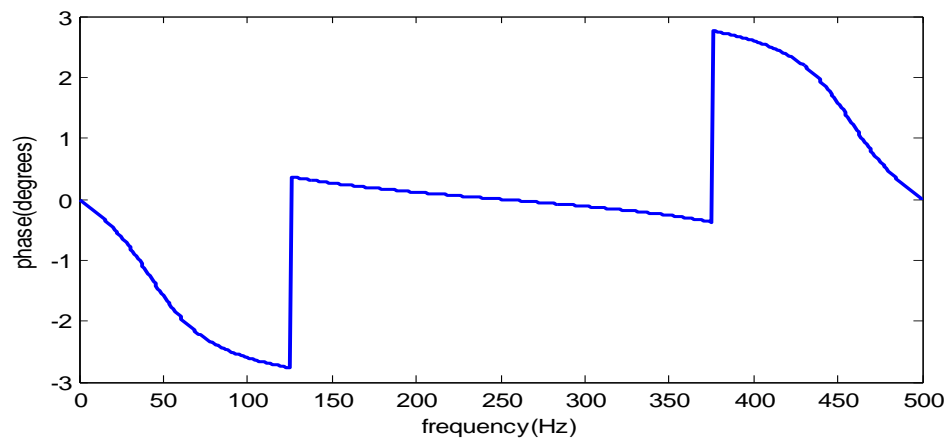


Figure 5: Phase plot of bandpass filter generated using BLT.

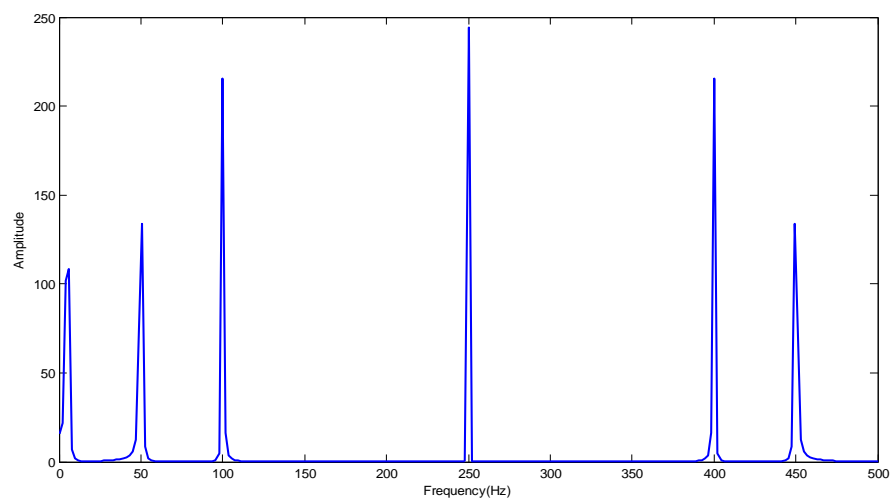


Figure 6. FFT of test signal.

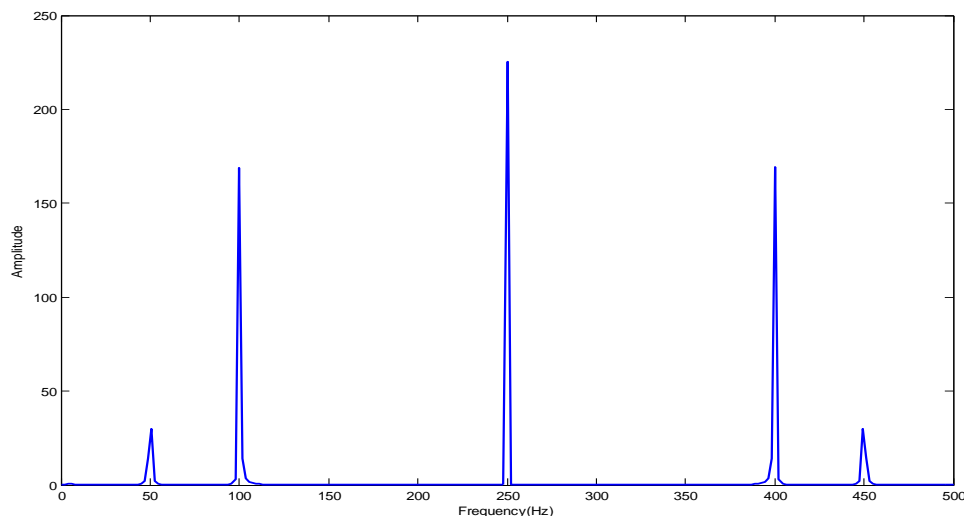


Figure 7. FFT of PSO designed BPF's response to test signal.

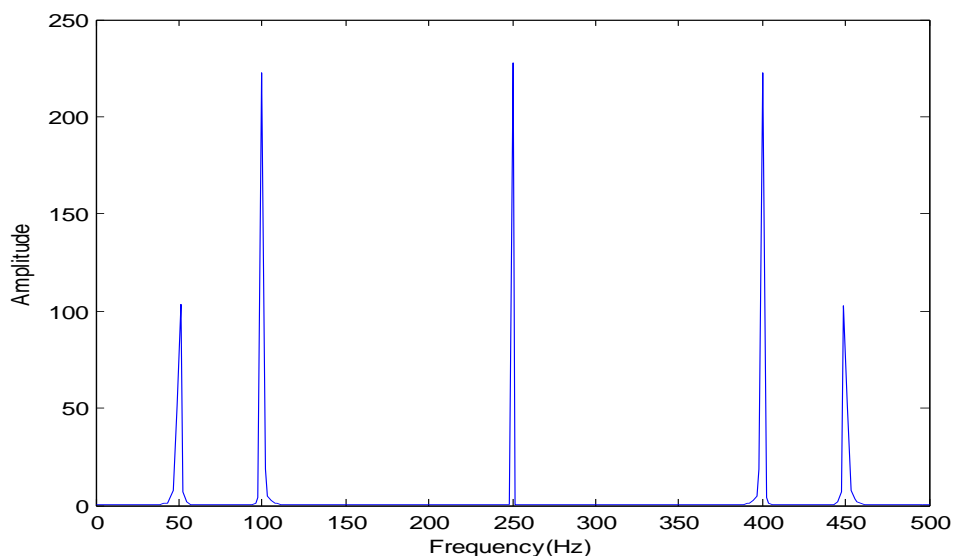


Figure 8. FFT of BLT designed BPF's response to test signal.

CONCLUSION

The classical bilinear transform (BLT) filter design technique and particle swarm optimization technique were applied to design the specified bandpass filters. Responses of the filters designed with both methods show good performance in discrimination against unwanted frequency bands.

Results of the simulated tests carried out on the filters conform to the given filter specifications. Magnitude responses of the designed filters compared with the ideal filter response clearly demonstrated the efficacy of PSO method in digital filter design. It is pertinent to note that the response of the filter designed using PSO satisfies the given specifications. Further work on hardware implementation of particle swarm optimization in digital filter design is an area of future focus.

References

- Adel M, Sharaf A A, EL-Gammal (2009). Optimal Hybrid Power Filter Compensator Design Using Multi – Objective Particle Swarm Optimization (MOPSO)", UKSIM, Pp. 391 – 397. UKSIM 2009: 11th International conference on computer modeling and simulation.
- Bipul L (2009), **Application Of Swarm, Evolutionary And Quantum Algorithms In System Identification And Digital Filter Design**: Msc. Thesis Missouri University of Science and Technology, p.p. 25 – 35.

- Bipul L, Venayagamorthy G K (2008), **Differential Evolution Particle Swarm Optimization For Digital Filter Design**: 2008 IEEE congress on Evolutionary Computation, pp. 3955 – 3960.
- Del Valle Y, Ganesh Kumar Venayagamoorthy, Salman Mohagheghi, Jean-Carlos Hernandez, Ronald G Harley (2008). Particle Swarm Optimization: Basic Concepts, Variants and Applications in Power Systems, IEEE Transactions on Evolutionary Computation, 12 (2) : 171 – 195
- Kazhar K, Khaled E M (2006). **Rational Based Particle Swarm Optimization For Digital Image Interpolation**: Inter. J. Intelligent systems. Technol. Pp. 223 – 226.
- Li Tan (1999). **Digital Signal Processing Fundamentals And Applications**: Elsevier, USA. Pp 314-333
- Omizegba C, Monikang A Fidelis, Ahmed O D (2010). **Non Linear Parameter Identification Using Particle Swarm Algorithm**: In proceeding of J. sustainable technol. 1(1): 85 – 91
- Swagatam D (2005) **An Efficient Evolutionary Algorithm Applied To The Design Of Two Dimensional IIR Filters**: Electronics and Telecom department, Jabalpur University, Kolkata 700032, India.
- Ying Z, Zeng Guangjie, Yu Feihong (2003). **Particle Swarm Optimization Based Approach For Optical Finite Impulse Response Filter Design**: Optical Society of America, OCIS codes, 120.2440, 120.4640, 260.1440, 350.2460. Pp 1503-1507